JUN 13 1963

144

N63-18869

ITEM

PROGRAM MANUAL

By

Fred Shaffer
R. K. Squires
Henry Wolf

Goddard Space Flight Center
Greenbelt, Maryland

TM X-50,131

ITEM
PROGRAM MANUAL

by
Fred Shaffer
R. K. Squires
Henry Wolf*

Goddard Space Flight Center
Greenbelt, Maryland

A program of this nature has developed over a period of several years; it is thus impossible to mention all those who have contributed. It was originally conceived by S. Pines and H. Wolf at Republic Aviation Corporation under contract to NASA (NASW-109) beginning in 1959. Major contributors have been C. Bergren, C. Hipkins, L. Lefton and M. Wachman.

Numerous additions and improvements have been made to the current version including reprogramming for the IBM 7090 and 7094 computers, and the development is a continuing effort.

The program has been and is available for general use for interested organizations.** Furthermore, a companion program is under development, at the instigation of the Manned Spacecraft Center, to rewrite the program in fortran, and subroutine form.

The authors express their appreciation to Mrs. D. Pollack and Miss J. Chiville for their assistance in the final preparation of this manual.

---

* Contractor with Analytical Mechanics Associates, Inc.

** Address: Head, Special Projects Branch, Theoretical Division
Goddard Space Flight Center, Greenbelt, Maryland.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

TABLE OF CONTENTS (Cont.)

TABLE OF CONTENTS (Cont.)

APPENDICES | Page

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

This report describes a general purpose Interplanetary Trajectory Encke Method (ITEM) Program, programmed for the IBM 7090 and IBM 7094. The method employed is designed to give the maximum available accuracy without incurring prohibitive penalties in machine time. On the basis of research described in reference 1, the Encke method was selected as best satisfying these requirements. However, the classical Encke method was modified to eliminate some of its objectionable features. This modified Encke method is described in the appendix.

The perturbations included in this program are the gravitational attractions of earth, moon, sun, jupiter, venus and mars considered as point masses. Additionally, the effects of the 2nd, 3rd and 4th zonal harmonics and 2nd tesseral harmonic of the earth's gravitational field as well as the triaxial lunar potential, aerodynamic drag, small corrective thrusts and radiation pressure, including the shadow effect of the earth are considered. The input may be prepared in any one of several common systems and a great variety of output options is available.

II. NOTATION

Upper case - vectors; Hats - unit vectors; Lower case - magnitudes

Description	Symbol	Units
Cartesian coordinates of vehicle with respect to reference body	x y z	km
Velocity components in Cartesian coordinates	$\dot{x}$ $\dot{y}$ $\dot{z}$	km/sec
Time	t	hrs.
Longitude measured from Greenwich, + East (used in Section IV and Appendix H)	θ	degrees
Longitude of vernal equinox	θ_0	degrees
Speed	v	km/sec
Geodetic altitude*	h	km
Geodetic latitude	ϕ	degrees
Geodetic flight path angle	γ	degrees
Geodetic flight path azimuth	A	degrees
Acceleration parameter (defined in Appendix E)	u	
Right ascension	RA	degrees
Astronomical units	AU	
Earth radii	ER	
Earth mass	m_e	

*Note: The following 3 symbols with primes denote the corresponding geocentric quantities. Geocentric in this report refers to a spherical earth, i.e., $e^2 = 0$. In this case $\phi' = \delta =$ declination.

Description	Symbol
Vehicle position vector	R
Distance to vehicle	r
Perturbation displacement vector	Δ R
Perturbation displacement vector components	ξ, η, ζ
Perturbation acceleration	F
Coordinate functions and their time derivatives respectively	$f, g, \dot{f}, \dot{g}$
Mass parameter	μ
Semi-major axis	a
Earth's eccentricity as used in Appendices H, I, L, S	e
Mean motion	n
Unit vectors for classical two-body orbit solution	$\hat{P}, \hat{Q}$
Eccentric anomaly as used in Appendix D	E
Elevation angle as used in Appendix I	E
$R_0 \cdot \dot{R}_0$	d_0
Incremental eccentric anomaly as used in Appendix D	$\theta = E - E_0$
Functions of θ defined by equations (E.2)	f_0, f_1, f_2, f_3, f_4
Inclination of orbital plane	i
Right ascension of the ascending node	Ω

Description	Symbol
Argument of perigee	ω
Parameters which account for polar oblateness of the earth, defined in Appendix H	c, s
Right ascension of the station meridian	RA_s
Range measured from observation station	ρ
Direction cosines measured in a topocentric coordinate system	λ, μ, ν
Declination	δ

SUBSCRIPTS

Vehicle	v
ith perturbing body	i
Quantity obtained from Keplerian solution of two-body problem	k
Reference body as used in Appendix B	c
Station	s
$R_A - R_B$	R_{AB}
Value at rectification time	o
Corresponds to x, y, z components respectively	n = 1, 2, 3
Value at perigee	p

III. GENERAL PROCEDURE FOR USING PROGRAMS

Initial conditions, terminal conditions and print frequency as well as other parameters controlling the flow of the program are read as input. The computation of the trajectory then proceeds until one of the terminal conditions (e.g. maximum time) has been reached or an error is encountered. At this time the program prints the reason for its termination and selected storage locations and proceeds to the next case. When an end of file is encountered on the input tape, control is transferred to the monitor.

In the case of a few error stops the on-line printer requests a core dump. However, most errors cause the case to terminate, and the computer proceeds to the next case. Non-monitor operation is described in detail in Section XII.

An option is also included, which permits the program to iterate on initial conditions for trajectories in order to satisfy a set of desired conditions at the moon, earth or T-planet.

IV. INITIAL CONDITIONS

The initial conditions necessary for the specification of a trajectory are:

1. Initial position of the vehicle relative to the reference body.
2. Initial velocity of the vehicle relative to the reference body.
3. Initial time of launch referenced to a base time. For the specification of the initial conditions the reference systems and units shown below may be used.

A. Cartesian Coordinates

The coordinate system is defined as follows:

1. The origin is at the center of the reference body.
2. The x-axis is in the direction of the mean equinox of date.
3. The xy-plane is the mean equatorial plane of the earth. Initial position is given by the x, y, z coordinates of the vehicle. Initial velocity is given by the $\dot{x}$, $\dot{y}$, $\dot{z}$ components of the velocity. Initial time of launch from base time[1] (t) is also given. If the program is used in its standard form the units[2] to be used for the above are:

$x\ y\ z$ – km

$\dot{x}\ \dot{y}\ \dot{z}$ – km/sec

t – days,[3] hours, minutes and seconds from base time

The year of launch must also be given.

---

(1) The base time is 0.0 h U.T. December 31 of the year previous to the year of launch.

(2) Scale factors are used to convert the input units to the units used internally (ER or AU and Hrs). Any other set of units may be used by changing these scale factors with a modification card as described in Section VIII.

(3) The number of days must be an integer (floating point), the minutes and seconds may be included in the number of hours as a fractional part. Zeros must then be loaded in place of minutes and seconds.

B. Geodetic Polar Coordinates

Initial position of the vehicle is given by:

1. Geodetic latitude (ϕ)

2. Longitude,[4] measured from Greenwich (θ)

3. Geodetic altitude (h)

4. Longitude of[4] vernal equinox at initial time (θ_0). This quantity may be computed by the program or may be loaded. (See IX-A Note 23)

Initial velocity of the vehicle is given by:

1. Speed (v) with respect to the center of the earth.

2. Flight path azimuth (A) measured clockwise from north in a plane normal to the geodetic altitude.

3. Flight path angle (γ) measured from a plane normal to the geodetic altitude.

Initial time of launch from base time (t) must also be given[1].

The following units must be used with the above:

1. ϕ, θ, and θ_0 - degrees; h - km.

2. A and γ - degrees; v - km/sec.

3. t-days, hours, minutes and seconds.

C. Geocentric Polar Coordinates

Ordinarily an input given in polar coordinates will be interpreted as described in paragraph B (preceding). However, if the eccentricity of the earth is set to zero, the program will interpret latitude as

---

[4] If the right ascension (RA) at initial time is known, it may be used in place of longitude (θ). The longitude of the vernal equinox (θ_0) is then set to zero.

declination, height as distance above a spherical earth of equatorial radius and flight path angle and azimuth with reference to a plane normal to the radius vector. If this interpretation of the input is desired the eccentricity of the earth $(e)^2$ is set to zero with a modification card, described in Section VIII-A. This eccentricity is used only in this part of the program. Therefore, other portions of the program will not be disturbed. An option is also included which interprets latitude and height as geodetic and azimuth and flight path angle as geocentric. (See VIII-C-1)

D. Polar Coordinates in Moon Reference

If polar coordinate input is used in moon reference the input quantities are interpreted in a righthanded coordinate system defined as follows:

1. The x y plane is the moon's orbital plane.

2. The x-axis points toward the earth.

3. The z-axis is parallel to the angular momentum vector of the moon about the earth.

In this system, latitude ϕ is

the angle between radius vector and x y plane, positive for positive z.

Longitude θ is

the angle between the projection of the radius vector on the x y plane and the x-axis, positive for positive y.

θ_0 irrelevant

Altitude h is

The distance from the surface of a spherical moon.

Speed v is

the speed with respect to the center of the moon in a nonrotating coordinate system.

Flight path azimuth and angle are defined in a plane normal to the radius vector.

E. Translunar Plane

The translunar plane input (See Figure 1) uses the initial conditions (height, h, speed, v, argument of radius Ψ (5)and flight path angle γ) with respect to the translunar plane. The inclination i_{TL} and the lead angle ϕ (6) serve to position the translunar plane with respect to the moon's orbital plane. The moon's orbital plane is related to the standard coordinate system by means of the ephemeris as described in Appendix V.

The translunar plane (See Figure 1) input consists of the following:

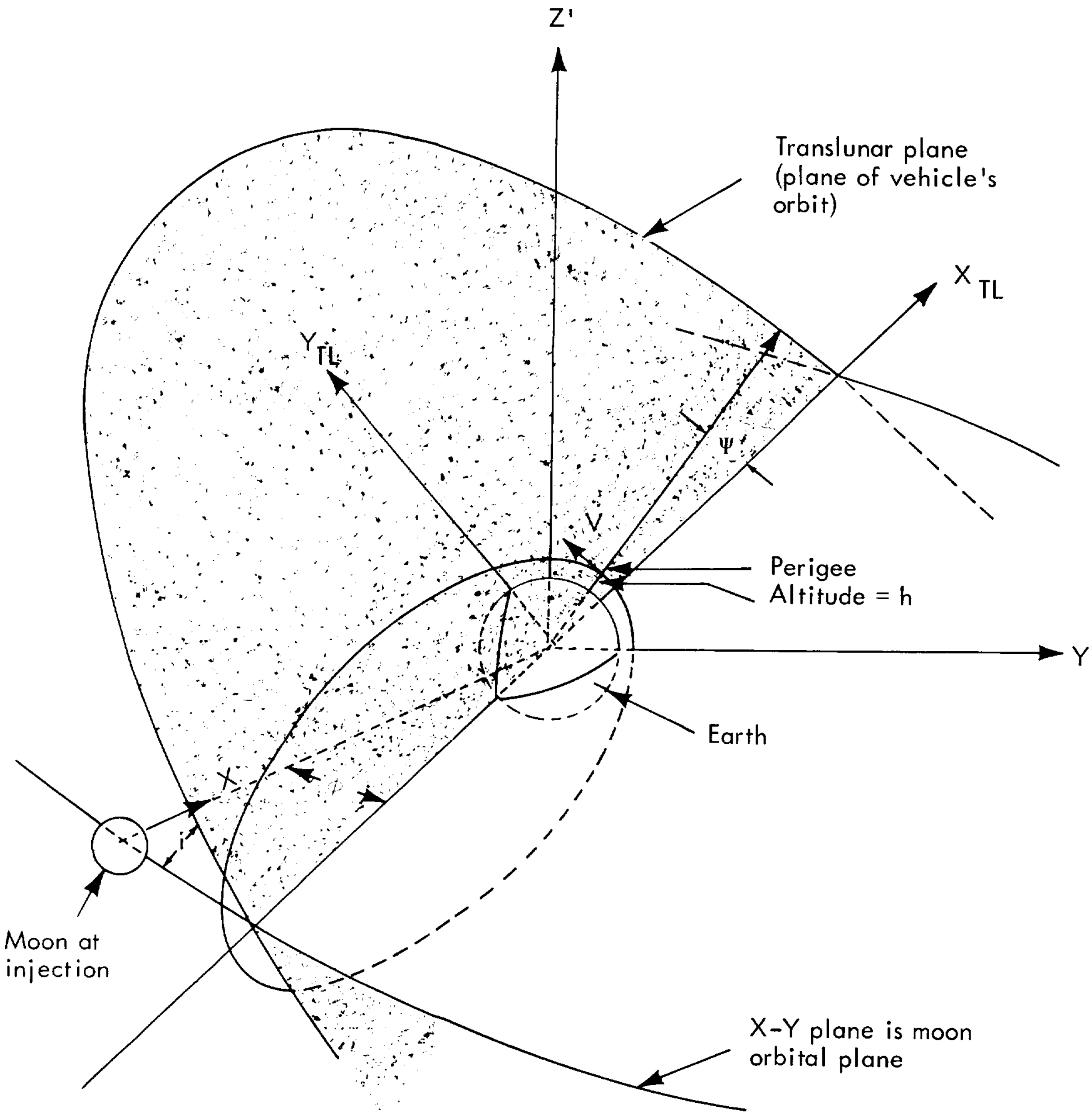
Load - 1	= Lead angle (ϕ) (degrees)
Load	= Case No.
Load + 1	= Argument of radius (Ψ) (degrees)
Load + 2	= Altitude (kilometers)
Load + 3	= Speed (km/sec)
Load + 4	= Inclination of plane (degrees)
Load + 5	= Flight path angle (degrees)

The remainder of the load is the normal polar load.

---

(5) The angle between the initial position vector and the ascending node wrt.the moon's orbital plane.

(6) The lead angle is the angle between the position vector of the moon at launch and the descending node of the translunar plane wrt. the moon's orbital plane.



Program symbol:

ϕ = Lunar lead angle
i = Inclination of translunar plane
Ψ = In-plane angle of radius = argument of radius
γ = Flight path angle, angle above local horizontal (geo or lunicentric)
h = Injection altitude

Figure 1. Translunar Plane

F. Trajectory Search

The search program is used in connection with the polar load and is activated in two ways: (see card format Section VIII)

1. If the search consists of circumlunar trajectories with return to earth –

   12166 ITENT 5238 OCT 002000060001 TRA STIT

   If lunar trajectories only are desired, the following modification card must also be added –

   60164 REPAR-1 24692 OCT 002000060211 TRA PREND

2. If the search is on T-planet either mars or venus –

   12167 VITENT 5239 OCT 002000060672 TRA VIT

The search input is also loaded through modification cards, sample of which follows:

62025 MAXIT	25621 DEC 5	(maximum number of iterations desired)*
62027 IVAR	25623 DEC 0,0,10.,.0001,0,0,0**	

Input quantities to be varied in the units and sequence of the polar load i.e., latitude, longtitude, altitude, velocity, azimuth, flight path angle and time, in the amounts to vary the inputs to get the partial derivatives. In the sample given, vary height 10 km, speed .0001 Km/sec., and keep all others constant. Next

62037 LQUANT 25631 DEC 5, 10*

the code of the quantities to be achieved. In the sample, pericynthion and perigee. The quantity code is as follows:

---

* Fixed point numbers
** Floating point numbers

1.	Lunar inclination	(degrees)
2.	Lunar ascending node	(degrees)
3.	Lunar argument of pericynthion	(degrees)
4.	Lunar time of pericynthion	(hours)
5.	Lunar pericynthion radius	(km)
6.	Earth inclination	(or T-planet)
7.	Earth ascending node	(or T-planet)
8.	Earth argument of perigee	(or T-planet)
9.	Earth time of perigee	(or T-planet)
10.	Earth perigee radius	(or T-planet)
11.	B·T - MISS PARAMETER	(T-planet or moon)
12.	B·R - MISS PARAMETER	(T-planet or moon)

Next

```
62051 QUANT        25641 DEC 6000., 5000.**
```

are the desired values of the dependent variables. These must be listed in the same order as LQUANT. Therefore, these are pericynthion and perigee radius (in km) respectively.

Finally, the tolerances on the above quantities should be given and again in the same order as the preceding two i.e., LQUANT and QUANT

```
62063 ICONV        25651 DEC 10.,100.**
```

also in km. Thus the tolerance on pericynthion radius is 10 km and 100 km for perigee. If the solution converges to within the specified tolerance, the iterations will stop.

** Floating point numbers

A maximum of any seven dependent variables can be selected. All of the search input modifications must be repeated for each repeat or stacked case with appropriate changes as desired.

If it is deemed adequate to keep the same matrix, at any point in the calculations, sense switch 3 may be depressed on the computer console. This considerably reduces the magnitude of the computer output as well as consuming less machine time, since, if a new matrix is required for each iteration, the nominal and all of the variational trajectories must be recomputed. It is, therefore, highly desirable to use the same matrix whenever possible. The matrix, residuals and changes in initial conditions are printed on line to permit following the progress and possibly to avoid time consuming computation.

To repeat — the sample input for search is as follows:

12166 OCT 002000060001	ACTIVATE SEARCH
25621 DEC 5	MAXIT*
25623 DEC 0,0,10.,.0001,0,0,0	IVAR**
25631 DEC 5,10	LQUANT*
25641 DEC 6000.,5000.	QUANT**
25651 DEC 10.,100.	ICONV**

G. Comments

1. The program computes in Cartesian coordinates.

The units used internally in the computation are:

a. position: Earth Radii (ER) - Astronomical Units (AU)

b. velocity: ER/hr. - AU/hr.

c. time : hrs.

(Earth radii units are used in the earth or moon reference. Astronomical units are used in the sun or T-planet reference.)

* Fixed point numbers

** Floating point numbers

2. Cartesian and polar coordinates can be used when launching from any body, providing ecliptic polar coordinates are used in sun or T-planet reference.

3. Equations for converting the initial conditions from polar to cartesian coordinates are shown in Appendix H.

V. TERMINATING CONDITIONS

The set of conditions which will terminate a trajectory may be summarized as:

1. Maximum time of flight (hrs.)

2. Maximum distance from any possible reference body considered in the solution.

3. Minimum distance from any possible reference body considered in the solution.

Any of these conditions will terminate a trajectory. Loading a large number into any of the maxima and a zero into any of the minima will make the corresponding conditions inoperative. A proper choice of these numbers will permit complete computation of the desired trajectory, avoid extensive unnecessary computation and guard against faulty input.

VI. PERMISSIBLE PERTURBATIONS

The trajectory computation consists of two parts, the exact solution to the two-body problem and integrated additions to this solution for the effect of perturbations. The successful control of round-off errors in the modified Encke method depends on preventing the accumulated round-off error in the integrated perturbation displacement from affecting the computed position. This is achieved by always keeping the perturbation displacement small by rectifying whenever the perturbation exceeds specified limits. The inputs determining the allowable limits are applied as a ratio of the perturbation position, velocity and an acceleration parameter to the two-body position, velocity and acceleration respectively.

This ratio is shown for the position vector in Figure 2.

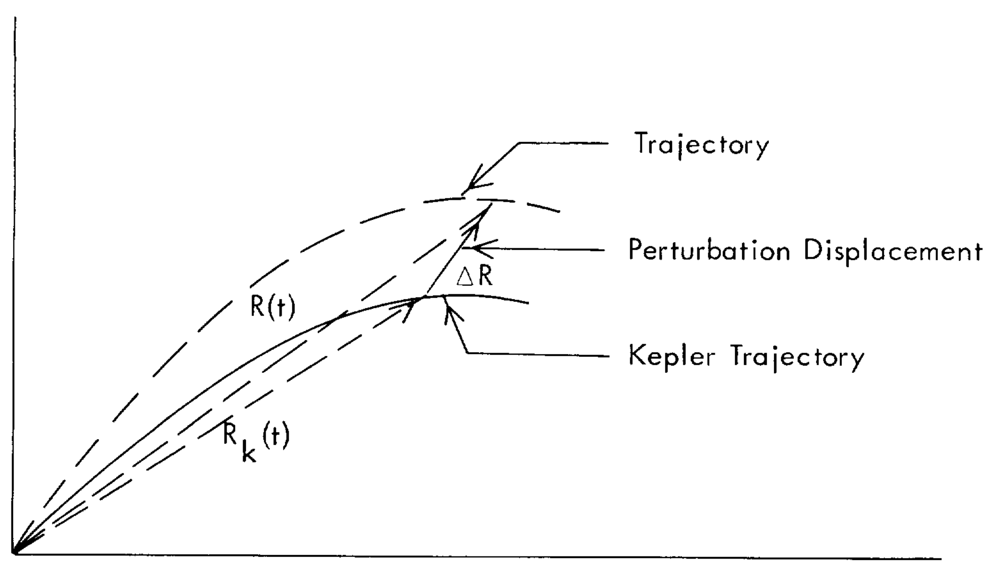


Figure 2. Encke Method

The recommended values for these ratios are as follows:

$$\frac{\Delta r}{r} \leq .01$$

$$\frac{\Delta \dot{r}}{\dot{r}} \leq .01$$

$$u^{*} \leq .01$$

* The parameter u is defined in Appendix E.

VII. RADAR INFORMATION PROGRAM

The program may be used to simulate radar data if desired. A maximum of 30 stations can be handled at one time. The following information is required for each station considered:

1. Station name - for identification purposes

2. Position of radar station

   a. Longitude (θ) of the station from Greenwich positive eastward in degrees, minutes and seconds. *

   b. Geodetic latitude (ϕ) of the station in degrees, minutes, and seconds, positive north. *

   c. Altitude (h) of station above sea level in feet.

The simulated radar information consists of azimuth, elevation, topocentric right ascension and declination, slant range and range rate. It is printed at every normal print time for which the elevation angle is positive. Refraction is not considered.

This part is coded as a subroutine and may be called at other times, if desired.

---

* Alternately these quantities may be given in degrees and decimals. 0's must be loaded into the positions reserved for minutes and seconds.

The fractional parts will not appear in the printout reproducing the station coordinates. They will, however, be included in the computation.

VIII. MODIFICATION CARDS

Normally, the program proceeds in the fashion described in Section IX. Modification cards may be used to operate the program in non-standard fashion to provide for special requirements. Any modification card included in a case will be operative for all succeeding cases in the input stack, unless it is revoked. Modification cards are listed in this report with their octal, symbolic and decimal locations. The octal and decimal locations refer to the "update 10" version of the program, and some may be different for later update versions. If such a later version is used, the octal and decimal locations and the addresses should be checked in the symbol table at the end of the assembly listing. The printout contains the update number used, following the title print. The decimal location is to be punched in columns 2-6, columns 8-10 contain one of the following operation codes

DEC, OCT, or BCD

A number to be loaded must begin in column 12 for DEC[1] and OCT cards. The first blank encountered to the right of column 12 terminates reading. A BCD card must have a word count[2] in column 12, blanks will not terminate reading of BCD cards.

Some modification cards which are frequently used are described below:

A. <u>Geocentric Polar Coordinates (i.e., spherical earth)</u>

Modification card

25507 IECC 11079 DEC 0*

(1) A DEC format will read in fixed or floating point numbers. A floating point conversion is performed if the number includes a decimal point or the symbol E.

(2) The number of words is the number of symbols (including blanks) divided by 6. A word count of 10 is assumed if column 12 is blank.

* Fixed point numbers

To revoke modification card:

25507 IECC   11079 DEC .006693422**

B. Cartesian Scale Factor – Input and Output

25445 REKM   11045 DEC ...   (1)

25446 MDKM   11046 DEC ...   (2)

25447 TSCL   11047 DEC ...   (3)

where

(1) Number of distance units/ER   **

(2) 23454.87 x (1) = Number of distance units/AU   **

(3) Number of time units/hour (used only if time scale is modified)   **

Example: Initial conditions to be in ER and ER/hr. Then card reads:

25445 REKM   11045 DEC 1.,23454.87,1.**   (1)

To revoke modification card:

25445 REKM   11045 DEC 6378.165,14.9599E7,3600.**

(1) Several numbers appearing on one card, separated by commas, are loaded into consecutive locations

C. To Initiate Radar

Modification cards (2 cards)

7426 RADSTA   3862 OCT 007400420742   TSX Z$BGN,4

The above card activates the loading of station coordinates

** Floating point numbers

11723 RADENT   5075 OCT 007400421114   TSX ZRADAR,4

The above card activates the computation of radar information.
To revoke modification cards (2 cards):

7426 RADSTA   3862 OCT 076100000000   NOP   (1)

The above card inactivates the loading

(1) This card must be included with the first repeat case if identical station locations are used.

11723 RADENT   5075 OCT 076100000000   NOP

The above card inactivates the computation

D. Table Print - Prints astronomical tables in core at every rectification.
Modification card

10331 TBPRNT   4313 OCT 007400423107   TSX TAPRN,4

To revoke modification card:

10331 TBPRNT   4313 OCT 076100000000   NOP

E. Table Print Exits
Normally the Program Continues after Table Print.

Modification cards

If desired to stop:

23117 TARTN   9807 DEC 0   HTR

To do next case:

23117 TARTN   9807 OCT 002000015245   TRA CCDS1

To revoke modification card:

23117 TARTN   9807 OCT 002000400001   TRA 1,4

F. <u>End Card</u>

Normally the Program Returns Control to the Monitor after all the cases are completed. If an "end of file" is desired on the output tape for non-monitor operation, the following card is added after the input.

37776 SWEF	TRA 16382	TRA SWEF

G. <u>The Normal Integration Interval in Moon Reference</u> is 1/16 of an hour and a normal print interval is 1 hour. If it is desired to change these, the following cards are added:

25544 MDET	11108 DEC ...	(New integration interval)**
25545 MPDT	11109 DEC ...	(New print interval)**

H. <u>Inclusion or Exclusion of Perturbations</u>

1. Ordinarily included are the gravitational field of the earth (2nd, 3rd and 4th zonal harmonics) and the gravitational attractions of sun, moon, mars, venus and jupiter. To exclude any or all of these perturbations, put 0 into the corresponding locations as follows:

25462 MS1	11058 DEC... **	Sun
25463 MM1	11059 DEC... **	Moon
25464 MP11	11060 DEC... **	Jupiter
25466 KMAR	11062 DEC... **	Mars
25467 KVEN	11063 DEC... **	Venus
25470 JOBL2	11064 DEC... **	2nd Zonal Harmonic
25471 JOBL3	11065 DEC... **	3rd Zonal Harmonic
25472 JOBL 4	11066 DEC... **	4th Zonal Harmonic

** Floating point numbers

2. The perturbation due to the Triaxial Potential of the moon is computed in moon reference. If it is to be omitted, put 0 into

25473 CONSC	11067 DEC 0*

3. The following numbers are to be loaded to restore standard operation of the program:

25462 MS1	11058 DEC ~~332488.~~ 332951.3	**
25463 MM1	11059 DEC .01229483	**
25464 MP11	11060 DEC 317.887	**
25466 KMAR	11062 DEC .1078210	**
25467 KVEN	11063 DEC .8147689	**
25470 JOBL2	11064 DEC .0323219422	**
25471 JOBL3	11065 DEC -.4579165E-4	**
25472 JOBL4	11066 DEC -.6719428E-4	**
25473 CONSC	11067 DEC .36366998E-2	**

The following perturbations are not ordinarily included:

4. Radiation Pressure may be included by loading a coefficient into

30133 RACOE	12379 DEC... **

The number to be loaded is:

$$\frac{KC_r A}{m}$$

* Fixed point numbers
** Floating point numbers

C_r is the radiation pressure in dynes/cm^2 at a distance of 1 AU from the sun.

$\left(C_r = 4.6 \times 10^{-5} \dfrac{\text{dynes}}{\text{cm}^2}\right)$ (Estimated value)

A area in cm^2

m mass in grams

K scaler $3600^2\ (23455.)^2\ /6378.165 \times 10^5 = .11178 \times 10^8$
sec to hrs, ER to AU, cm to ER

The radiation pressure will only be active if sunlight impinges on the vehicle. For correct results the radiation pressure should therefore be run only in conjunction with the optional shadow computation as described in Appendix O.

If, however, the expected trajectory may be safely assumed to be entirely out of the earth's shadow, shadow testing may be avoided, with a consequent saving in machine time. In this case the following modification card must also be included:

31553 SHADIN 13163 DEC 1. **

5. If inclusion of the Aerodynamic Drag is Desired, the drag parameter $1/2\ C_D\ A/m$ must be loaded in location COEFL by means of the following card:

25525 COEFL 11093 DEC ... **

The units of $C_D\ A/m$ are the area in cm^2 and the mass in grams. A layered atmosphere rotating with the earth is assumed. The density is obtained by a linear interpolation of density — altitude table.

6. To include equatorial oblateness, i.e., 2nd tesseral harmonic, load

$$\frac{3}{2}\ \mu\ J = 8.E\text{-}4 \qquad \text{(Extreme value)}$$

** Floating point numbers

into

25514 JEQ 11084 DEC 8.E-4 **

μ = 19.9094165 (g in program units).

7. If any of these perturbations are to be omitted in repeat cases, zero must be loaded in the corresponding locations.

In order to provide a permanent record of the perturbations included in each particular trajectory the numbers appearing in the locations described in Section VIII-H are printed at the beginning of each trajectory.

I. Atmospheric Tables For The Drag Computation are stored in core. They correspond to model #7, contained in Report #25 (Reference 2) of the Smithsonian Astrophysical Observatory, fitted to the ARDC Model Atmosphere of 1956 (Reference 3) at low altitudes. The units for the air density are grams/cm^3 and the height is given in ER from the center of the earth. If it is desired to change this atmosphere, the following procedure has to be followed:

Modification card

32500 NTAR	13632 DEC ...*	the number to be entered is N-1, where N is the number of entries in the density table.

Modification card

32501 RTBL	13633 DEC . . , . . . , . . ,**	the values of r at which the density is given in ascending order, a maximum of 50.

* Fixed point numbers

** Floating point numbers

Modification card

32563 RHOT	13683 DEC . . , . . . ,**	the values of the air density in gm/cm^3 in respective order corresponding to the r table

If other units are used for the density table the drag parameter described in Part H of this section has to be read in the same units and the constant (-6378.165E5)** normally in

32645 DRSC	13733 DEC... **	has to be changed accordingly.

The negative sign directs the drag force opposite to the velocity. This constant converts the drag from the units used for A, M, and ρ to ER/hr^2.

J. Angle From Ascending Node to Vehicle. To print angle add the following card:

23532 EAFANS 10074 OCT 002000033545 TRA ASCSAT

To revoke:

23532 EAFANS 10074 OCT 076100000000 NOP

K. The Program Provides a Special Printout described in the output (Section X) near the earth, moon, sun and T-planet. This printout occurs at every integration step and is useful to observe the behavior of these relevant quantities during ascent and re-entry. This feature is triggered by the following modification cards:

25534 SERE	11100 DEC...,**	earth reference	(ER)
25535 SMRE	11101 DEC...,**	moon reference	(ER)
25536 SSRE	11102 DEC...,**	sun reference	(AU)
25537 STRE	11103 DEC...,**	T-planet reference	(AU)

** Floating point numbers

The numbers used above are the radial distance within which the special print is to be effective. The units are earth-radii for the earth and moon references and astronomical units for sun and T-planet references. A zero in any of the above locations suppresses this feature.

L. The Rectification Print may be Eliminated With the Following Cards:

10206 MPH	4230 OCT 076100000000,076100000000	(1)
10211 EPH	4233 OCT 076100000000,076100000000	(2)
10227 SPH	4247 OCT 076100000000,076100000000	(3)
10241 TPH	4257 OCT 076100000000,076100000000	(4)
10751 RP	4585 OCT 076100000000,076100000000	(5)
10762 RECTPS	4594 OCT 076100000000	(6)

In order to obtain the rectification prints in repeat cases, after they have been eliminated in one case, the original contents, as given in the assembly listing of these locations have to be restored.

(1) Eliminates moon print heading

(2) Eliminates earth print heading

(3) Eliminates sun print heading

(4) Eliminates T-planet print heading

(5) Eliminates ratio print

(6) Eliminates normal rectification print

In order to get the normal print at t = 0 (which is also a rectification), the program tests the time. If zero, it prints normal output otherwise the skip print routine is effective.

M. To Compute and Print Latitude, Longitude and Velocity at Perigee add the following card:

23453 PGCOOV 10027 OCT 076100000000 NOP

N. Print at Ascending Node

To obtain the normal print at the crossing of the ascending node insert a fixed point number in:

25540 CCNT   11104 DEC...*

The number determines how often the print operates, i.e., 2 means print every second crossing of the ascending node. The print occurs at end of the first integration interval following the crossing.

O. Ordinarily the Printout Includes the Following:

Time, position with respect to reference body.
Velocity with respect to reference body.
More printout may be called for by putting a number (fixed point) into

25762 KM   11250 DEC...*

Restrictions. KM is between 1 and 8

1 in KM adds veh. position wrt earth

2 in KM adds veh. position wrt moon

3 in KM adds moon's position wrt earth

4 in KM adds veh. position wrt sun

5 in KM adds veh. position wrt T-planet

6 in KM adds perturbation displacement

7 in KM adds perturbation velocity

8 in KM adds perturbation acceleration

In addition printout will always contain

11463 LLDEC+2   4915 RA and declination

11661 EAZCAL-5   5041 Coordinates of subsatellite point

*Fixed point numbers

11715	1CONV+5	5069	Moon longitude and latitude in moon reference
11722	PREXT	5074	Osculating elements

Any of these prints may be avoided by appropriate coverings.

P. Osculating Elements Print is normally given at each print time.

To revoke:

11722 PREXT 5074 OCT 076100000000 NOP

To restore:

11722 PREXT 5074 OCT 007400423121 TSX ELCO,4

Q. The Normal Output Refers the Osculating Elements to the equatorial plane with the x-axis along the mean equinox of date.

The modification described below will trigger, in addition, the printing of the inclination, ascending node and argument of perigee with respect to the moon's orbital plane and the x-axis along the moon-earth vector, either at the running time t (see Figure 3) or at an arbitrary fixed time t_1. In addition, this modification will print the instantaneous osculating elements of the moon's orbit wrt the equatorial plane.

23527 ELPT 10071 OCT 002000033015 TRA INSMOO

To use orbital plane at time t_1:

23527 ELPT	10071 OCT 002000033000	TRA TOOSC+1
32777 TOOSC	13823 DEC ...**	time (1)

(1) If no number is loaded into TOOSC, the orbital plane at the initial time (t = 0) is used.

To revoke:

23527 ELPT 10071 OCT 076100000000 NOP

** Floating point numbers

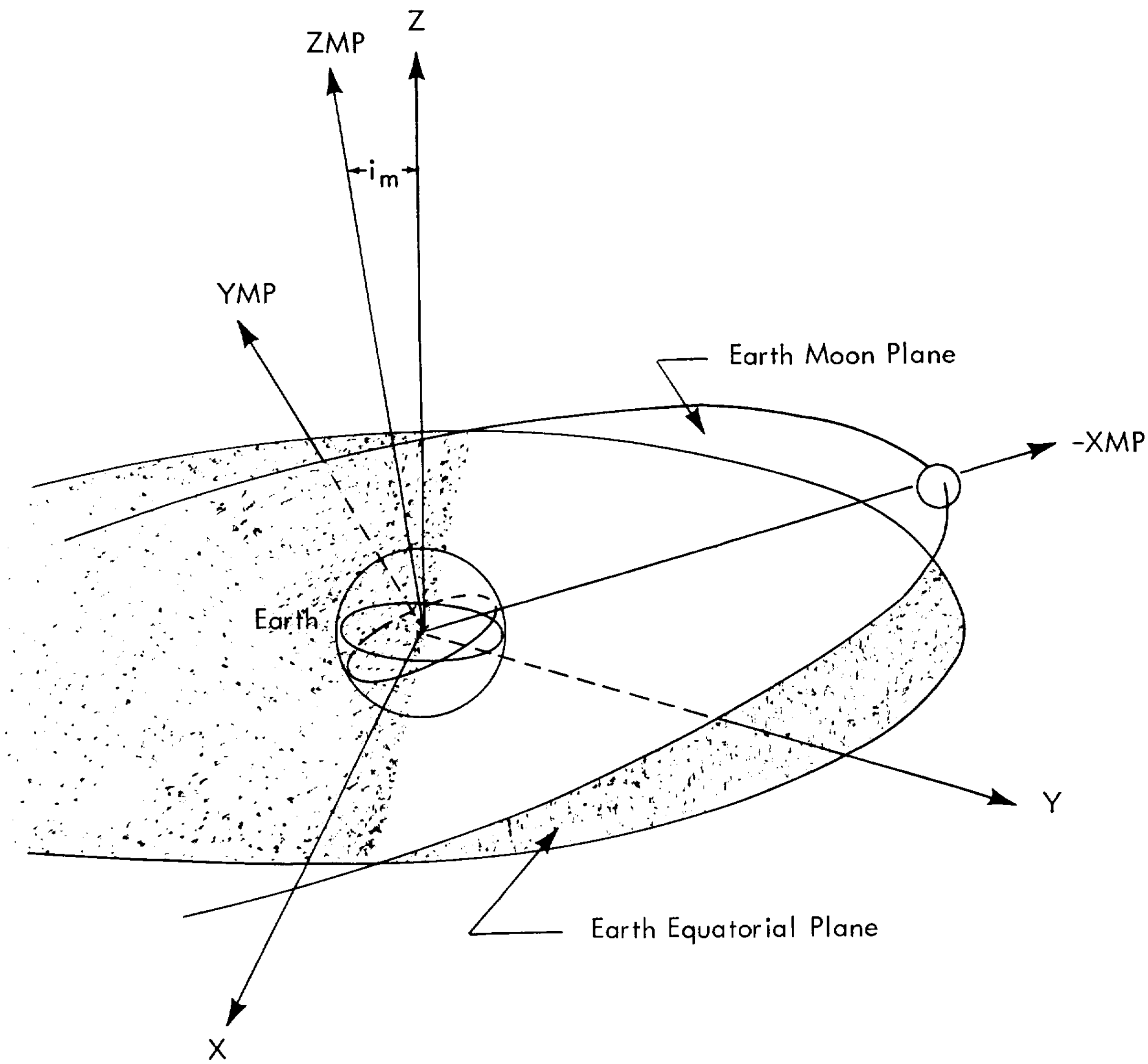


Figure 3. Instantaneous Lunar Plane Coordinates

R. Moon Fixed and Rotating Coordinate System

Vehicle position is provided in two coordinate systems, one rotating and one space fixed, based on the earth-moon plane. For both systems, the origin is at the center of the earth with the positive x-axis directed toward the center of the moon. The x y plane is the plane of the moon's orbit about the earth, and the z axis is in a northerly direction so as to form a right-hand system. For the rotating system (XROT, YROT, ZROT) the x-axis moves with the moon; for the space fixed system (XINJ, YINJ, ZINJ) the x-axis is directed toward the position of the moon's center at injection.

To use insert the following modification card:

23342 DONRCO	9954 OCT 076100000000	NOP

To revoke:

23342 DONRCO	9954 OCT 002000023422	TRA FOSCL

S. Perigee and Pericynthion Print

An option to print near perigee or pericynthion is included. The distance from the vehicle to reference body is tested, therefore, the print may be off as much as two integration intervals. When using this option all other print should be suppressed by using very large print intervals and the rectification print suppress.

To use the following modification card:

12647 STBLK-1	5543 OCT 002000012475	TRA PPGPT

To revoke:

12647 STBLK-1	5543 OCT 076100000000	NOP

T. To Print the Coordinates of the Earth Sub-satellite point and right ascension and declination in other than earth or moon reference, the following card must be added:

Modification card:

11441 NESS	4897 OCT 002000011443	TRA * +2

To revoke:

11441 NESS	4897 OCT 002000011667	TRA SUBSEX

U. Floating Point Spill Print

A routine is included which will make the proper correction for floating point spills. Normally this routine will print as a warning

OVERFLOW IN LOC.......

To eliminate print use the following card:

22736 FPSP	9694 OCT 002000022740	TRA * +2

To restore

22736 FPSP	9694 OCT 007400403746	TSX E$WOT,4

Overflow Diagnostic

This optional debugging feature prints at each overflow the contents of the index registers, AC, MQ, the address part of the three instructions preceding the overflow and the numbers in the listed locations.

To activate, use the following card:

22740 OFLDG	9696 OCT 076100000000	NOP

To revoke:

22740 OFLDG	9696 OCT 002000023002	TRA LLXT

V. Error Return

If an error is detected the program has three options

1. ERPRT — The octal location of the error is written on line and off line. The normal end dump is written on A3 and the program goes to the next case.

2. ERPRTS — The program behaves as above except that it requests a memory dump and halts.

3. ERPRTC — The program behaves as in ERPRT except that it will continue the current case through 3 of these errors before going on to next case.

To halt and request a dump after any error return include

13120 EREX-1	5712 OCT 002000040000	TRA PDUMP

W. The Flight Path Angle and azimuth referred to both the geodetic and geocentric horizontal plane in earth reference are printed normally.

To eliminate:

11666 EAZCAL	5046 OCT 076100000000	NOP

To revoke:

11666 EAZCAL	5046 OCT 007400434525	TSX AZZCAL,4

In moon reference

To eliminate:

11717 MAZCAL	5071 OCT 076100000000	NOP

To revoke:

11717 MAZCAL	5071 OCT 007400434525	TSX AZZCAL,4

X. Additional Stopping Conditions were introduced for moon satellites when the radius from the moon is too large, or too small. These distances may be changed by the following cards:

Modification card:

25440 MMIN	11040 DEC ...**	Min dist. (.27 ER currently)
25441 MMRAD	11041 DEC ...**	Max dist. (1E20 ER currently)

** Floating point numbers

Y. Shadow Calculation

The time boundaries defining the umbra and penumbra are determined (including refraction) by a linear interpolation across the two nearest integration steps. The radiation pressure switch is controlled only to the first integration step after a change occurs. Subsequent updates will include an optional refinement to integrate exactly to the boundary.

The entry, exit, time in and accumulated time for sunlight, umbra and penumbra are printed at each change. The following card activates shadow testing and printing:

```
12270 SHAENT    5304 OCT 007400423676    TSX WSHADE,4
```

To revoke:

```
12270 SHAENT    5304 OCT 076100000000    NOP
```

Z. Polar Scale Factor used to change the units (for polar input only) the modification card is:

```
25504 ICONT     11076 DEC ...            (1)**

25505 POLSC     11077 DEC ...            (2)**
```

(1) Number of time units in one hour.

(2) Number of distance units in one earth radius.

To revoke:

```
25504 ICONT     11076 DEC 3600.,6378.165**
```

A-1. Ecliptic Coordinate Output

This gives additional output (Section X-M) referenced to the ecliptic plane.

** Floating point numbers

To initiate include:

23530 ECELPT   10072 OCT 002000033604   TRA ECLCOO

To revoke:

23530 ECELPT   10072 OCT 076100000000   NOP

B-1. Impact Parameter Calculation (Appendix U)

This calculates miss distances on the impact plane.

To use include:

6753 TIMPPS   3563 OCT 076100000000   NOP

To revoke:

6753 TIMPPS   3563 OCT 002000006772   TRA ETIMPP

If several cases are stacked the same impact plane is used for all of them. If a new impact plane is desired, include the card:

34372 NOMTR   14586 DEC 0

C-1. Geocentric Velocity and Geodetic Position Input

To use include:

7257 TGEOCV   3759 OCT 002000007341   TRA GEOCV

To revoke:

7257 TGEOCV   3759 OCT 076100000000   NOP

D-1. Trajectory Search

The normal input for the trajectory search is by modification cards as given in Section IV-F. The lunar search option normally iterates on osculating elements referred to the equatorial system. To iterate on elements referred to the earth-moon plane use the following card:

```
60062 EMP6      24626 OCT 076100000000    NOP
```

To revoke:

```
60062 EMP6      24626 OCT 002000060103    TRA EQCOO
```

E-1. Ephemeris Time

The planetary coordinates are interpolated using ephemeris time

$$ET = UT + \Delta T$$

An approximate value of ΔT (35 sec) is used. To change, use the following card, giving ΔT in hours.

```
25502 ETMUT     11074 DEC ...**                (ΔT IN HRS)
```

To revoke:

```
25502 ETMUT     11074 DEC .009888888889** (ΔT IS 35 SEC)
```

F-1. If it is desirable to include additional descriptive material in the title printout,

The following locations may be used:

15716 TTPRN	7118 BCD	Descriptive information
15730	7128 BCD	Descriptive information
		20 symbols (2 cards) max.

The first letter of the first card should be in column 14.
The first letter of the second card may be in column 13.

** Floating point numbers

IX. INPUT

This section contains the following information:

A. General inputs necessary for running a case.

B. Stacking cases

C. Sample input for polar coordinates, no modifications

D. Sample input for Cartesian coordinates, no modifications.

E. Sample input with modifications

A. General Inputs

Title and Units	See Note
Modification Cards, if any	(1)
TRA 2,4	(2)
DEC 0, 1, 2 or 3	(3)*
0 - Cartesian coordinate input	
1 - Equatorial polar coordinate input	
3 - Translunar plane input	
3 - Ecliptic polar coordinate input	
DEC 0 or 1	(4)*
0 - Venus is T-planet	
1 - Mars is T-planet	

* Fixed point numbers

	Cartesian Coordinates	Polar Coordinates	Translunar Plane	See Note
-1	Omit	Latitude-Degrees and Decimals	Lead angle ψ Degrees and Decimals	(3)**
0		Case number		(5)*
1	x-Km	Longitude-Degrees and Decimals	Argument of Radius Degrees and Decimals	(3)**
2	y-Km	Altitude-km	Altitude-km	(3)**
3	z-Km	Speed-km/sec	Speed-km/sec	(3)**
4	$\dot{x}$-Km/sec	Flight Path Azimuth Degrees and Decimals	Inclination of Trans-lunar Plane-Degrees and Decimals	(3)**
5	$\dot{y}$-Km/sec	Flight Path angle Degrees and Decimals	Flight Path angle Degrees and Decimals	(3)**
6	$\dot{z}$-Km/sec	Longitude of Vernal Equinox-Degrees and Decimals or indicator	Immaterial	(3)(23)**

* Flight point numbers

** Floating point numbers

Card Number	Title and Units	See Note
7	Launch time – days	(8)(6)**
8	Launch time – hours	(8)**
9	Launch time – minutes	(8)**
10	Launch time – seconds	(8)**
11	Initial print suppress — hours	(7)**
12	Year of launch	(25)*
13	Initial reference – origin indicator	(3)(9)*
	1 – Earth 2 – Moon	
	3 – T-Planet 4 – Sun	
14	Thrust indicator	(10)*
	0 – No thrust	
	1 – Thrust	
15	Maximum time of flight – hours	(11)**
16	Maximum distance from earth-ER	(11)**
17	Minimum distance from earth-ER	(11)**
18	Maximum distance from T-Planet-ER	(11)**
19	Minimum distance from T-Planet-ER	(11)**
20	Maximum distance from sun-ER	(11)**
21	Minimum distance from sun-ER	(11)**

* Fixed point numbers
** Floating point numbers

Card Number	Title and Units	See Note
22	Position rectification constant	(12)**
23	Velocity rectification constant	(12)**
24	Acceleration rectification constant	(12)**
25	Integration interval-near earth-hours	(13)(24)**
26	Integration interval in earth reference but not close – hours	(14)(24)**
27	Integration interval in sun reference – hours	(15)(24)**
28	Integration interval in T-Planet reference but not close – hours	(16)(24)**
29	Integration interval close to T-Planet – hours	(17)(24)**
30,31,32,33	BCD 4 – Title or other identifying information for print	(5)(18)
34	Print interval – near earth – hours	(13)(19)**
35	Print interval – in earth reference but not close – hours	(14)(19)**
36	Print interval in sun reference – hours	(15)(19)**
37	Print interval in T-Planet reference but not close – hours	(16)(19)**
38	Print interval near T-Planet – hours	(17)(19)**

** Floating point numbers

If thrust is used, the number 1 has to be loaded into "Load + 14" and additional input data have to be supplied. These cards are inserted following the regular input, but preceding the radar input, if any. The thrust load has the following format:

T0	Integration interval to be used during thrust period	(24)**
T1	Number of thrust periods, 24 maximum	(26)*
T2	Initial mass of vehicle - pounds	(27)**
T3	Mass flow-pound mass/pound thrust/hour	**
T4	Initial time of first thrust period	**

For each thrust period: (28)

1T1	T_x :x - Component of thrust - pounds	**
1T2	T_y :y - Component of thrust - pounds	**
1T3	T_z :z - Component of thrust - pounds	**
1T4	0,0 - Space reserve	
1T5	t_F - Termination time of this thrust period	**

If simulated radar information is desired the following input must be supplied: (see VIII-C.) (20)

R0	Number of stations	(21)*
R1	BCD 4 - Station name	(5)(22)
R2	Longitude - degrees, minutes, seconds	(20)(22)**
R3	Latitude - degrees, minutes, seconds	(20)(22)**
R4	Altitude - feet	(20)(22)**

* Fixed point numbers
** Floating point numbers

NOTES:

1. See Section VIII for modifications available.

2. Terminates reading of modification cards. This card must always be included.

3. See Sections IV and VIII and translunar plane section.

4. The choice of the T-planet is significant only if trajectories are to be computed which will come close to either mars or venus. In this case, the planet designated may serve as a reference body if its sphere of influence is entered. For all other trajectories the designation of a T-planet only affects the output and is usually unimportant. A choice, however, must be made.

5. These inputs are used only for identification and not for computing purposes.

6. Number of days since December 31, 0.0 hrs UT. (See footnote 1 of Section IV.)

7. This suppresses printing until after specified time has been exceeded. The input time is arbitrary and may be chosen as desired.

8. The number of days must be a floating point integer, the minutes and seconds may be included as a decimal fraction of hours. In this case 0 has to be entered on cards 9 and 10.

9. Indicates the body whose center is the origin of the initial reference system.

10. For a more detailed description of the use of the thrust programs, see Section IX-A, B (Input).

11. See Section V.

12. See Section VI.

13. Within 4 ER   Recommended interval   .015625-.0625 hour

NOTES (cont)

14.	Between 4-123 ER	Recommended interval	.5-3 hours
15.		Recommended interval	12-24 hours
16.	Between 4-80 ER	Recommended interval	.5-3 hours
17.	Within 4 ER	Recommended interval	.06250 hour

18. Other identifying information desired.

19. Any interval desired may be used but it must be no less than the corresponding integration interval.

20. See Section VII.

21. Enter number of tracking stations.

22. Enter the following information for each station: (See Section VII).

BCD 4	station name 24 symbols maximum
DEC	longitude
DEC	latitude
DEC	altitude

Longitude and latitude may be given in degrees and decimals if more convenient, 0 then has to be used in place of minutes and seconds. The resulting printout of the polar station coordinates will then show the integral number of degrees only. The station position in core, however, will reflect the fractional part.

23. If zero is loaded in this position, the load in position 1 "longitude" is interpreted as right ascension. If a fixed point 1 is loaded, the program computes the longitude of the vernal equinox at launch time from the data supplied on cards 7, 8, 9 and 10. If for some reason this computation is not desired, a floating point number is loaded on card 6 and then this number will be used as the longitude of the vernal equinox.

NOTES (cont)

24. If fractional parts of hours are used for integration intervals, it is recommended that multiples of negative powers of 2 are used to eliminate round-off error in the time.

25. This quantity ensures that the proper file on the planetary tape is used. It will stop the program if the wrong tape is mounted or the proper file cannot be found.

26. Periods of no thrust intervening between thrust periods must be included in this count.

27. No staging is considered, but may be included if required.

28. For a period of no thrust intervening between two thrust periods cards 1T1 through 1T4 have to be included with 0 in place of T_x, T_y and T_z. In each case the final time t_F is used as initial time for the next thrust period, until the indicated number of thrust periods has been reached.

B. Stacking of Cases

If many cases are to be run differing only by a few parameters, it is not necessary to repeat that part of the input which remains unchanged. The changes only are to be loaded but a decimal location must be punched in columns 2 through 6. For instance, if only the latitude is to be changed, the following card is loaded:

26710	11720 DEC . . .	latitude

The table on the following page gives a complete list of the load, their octal locations and the decimal equivalent.

If the launch time is changed in a polar load stack the computation of the longitude of the vernal equinox must be re-initiated by loading 1 into LOAD + 6.

These cards are followed by 3 TRA 2,4 to terminate loading. If radar is used and the station coordinates are unchanged an "Avoid Radar Load" card (described in Section VIII-C) has to be included. If thrust is used, the thrust load has to be repeated or

TABLE 1 — LOAD LOCATIONS

LOAD NO	DEC	OCT	
-1	11720	26710	latitude
0	1	1	case no
1	2	2	x or long
2	3	3	y or height
3	4	4	z or speed
4	5	5	$\dot{x}$ or azimuth
5	6	6	$\dot{y}$ or flight path
6	7	7	$\dot{z}$ or l.v.e.
7	8	26720	days
8	9	1	hrs
9	11730	2	min
10	1	3	secs
11	2	4	print suppress
12	3	5	year
13	11734	26726	orig. indicator
14	5	7	thrust indicator
15	6	26730	max time
16	7	1	max dist earth
17	8	2	min dist earth
18	9	3	max dist run
19	11740	26734	min dist sun
20	1	5	max dist T-planet
21	2	6	min dist T-planet
22	3	7	pos rect const
23	4	26740	vel rect const
24	11745	1	acc rect const
25	6	2	integration interval near earth
26	7	3	integration interval far earth
27	8	4	integration interval sun
28	9	5	integration interval far T-planet
29	11750	6	integration interval near T-planet
30	1	7	identifying information
31	2	26750	
32	3	1	
33	4	2	
34	5	3	print where near earth
35	6	4	far earth
36	7	5	sun
37	8	6	far T-planet
38	11759	26757	near T-planet

the following modification card has to be included to cancel the thrust loading:

```
7415 ITHLD    3853 OCT 076100000000    NOP
```

To revoke:

```
7415 ITHLD    3853 OCT 007400415347    TSX THLD,4
```

C. Sample Input for Polar Coordinates — with repeat case

11084	DEC 8.E-4	JEQ 25514 Modification to include Equatorial Oblateness	
7118	BCD	FRED SHAFFER	
	TRA 2,4	Go to regular load — end of mod cards	
	DEC 1	Polar coordinates	
	DEC 1	Mars is T-planet	
	DEC 30.60860	Latitude in degrees and decimals	-1
	DEC 1	Case number	0
	DEC -71.78139	Longitude in degrees and decimals	1
	DEC 160.72594	Altitude in kilometers	2
	DEC 7.8439280	Velocity in km/sec	3
	DEC 77.89589	Flight path azimuth in degrees and decimals	4
	DEC -.0317	Flight path angle in degrees and decimals	5
	DEC 1	Program computes longitude of vernal equinox at launch time	6
	DEC 268.	Launch time in days from Dec 31,0.0h UT	7

DEC 14.	Launch time in hours from mid-night UT	8
DEC 0.	Launch time in minutes	9
DEC 0.	Launch time in seconds	10
DEC 0.	Print suppress in hours	11
DEC 1962	Year of launch	12
DEC 1	Initial ref. origin indicator 1EO, 2MO, 3TO, 4SO	13
DEC 0	0 No thrust, 1 thrust, thrust indicator	14
DEC 24.	Max. time of flight in hours	15
DEC 2.	Max. dist. from earth in ER	16
DEC 1.	Min. dist. from earth in ER	17
DEC 1E10	Max. dist. from T-planet mars in ER	18
DEC 1.	Min. dist. from T-planet mars in ER	19
DEC 1E10	Max. dist. from sun in ER	20
DEC 1.	Min. dist. from sun	21
DEC .02	Position rect. const.	22
DEC .02	Velocity rect. const.	23
DEC .02	Acceleration rect. const.	24
DEC .03125	Integ. interval near earth (hours)	25
DEC .25	Integ. inter in earth ref, not close (hours)	26

	DEC 24.	Integration inter in sun ref (hours)	27
	DEC 24.	Integ inter in T-planet ref, not close (hours)	28
	DEC 24.	Integ inter near T-planet (hours)	29
	BCD 4 MA-7	Polar load (title)	30-33
	DEC .25	Print inter near earth (hours)	34
	DEC .5	Print inter in earth ref, not close (hours)	35
	DEC 24.	Print interval in sun ref (hours)	36
	DEC 24.	Print inter in T-planet ref, not close (hours)	37
	DEC 24.	Print inter near T-planet (hours)	38
11084	DEC 0	No equatorial oblateness in repeat case	
7128	BCD	NO EQUATORIAL OBLATENESS	
	TRA 2,4		
	TRA 2,4		
	TRA 2,4		
	TRA 16382	End of run, non monitor	

D. Sample Input for Cartesian Coordinates, no Modifications

TRA 2,4	
DEC 0	Cartesian coordinates
DEC 0	Venus is T-planet

DEC 1	Case number	0
DEC 4953.3445	x coordinate km	1
DEC 3716.7623	y coordinate km	2
DEC 2224.0628	z coordinate km	3
DEC 7.1423107	x vel component km/sec	4
DEC 7.6362243	y vel component km/sec	5
DEC 3.1457277	z vel component km/sec	6
DEC 33.	Launch time in days	7
DEC 2.	Launch time in hours	8
DEC 59.	Launch time in minutes	9
DEC 42.72	Launch time in seconds	10
DEC 0.	Print suppress in hours	11
DEC 1966	Year of launch	12
DEC 1	Initial ref. origin indicator 1EO, 2MO, 3TO, 4SO	13
DEC 0	0 no thrust, 1 thrust, thrust indicator	14
DEC 168.	Max time of flight in hours	15
DEC 70.	Max dist from earth in ER	16
DEC 1.	Min dist from earth in ER	17
DEC 1E10	Max dist from T-planet mars in ER	18
DEC 1.	Min dist from T-planet mars in ER	19

DEC 1E10	Max dist from sun in ER	20
DEC 1.	Min dist from sun	21
DEC .02	Position rect const	22
DEC .02	Velocity rect const	23
DEC .02	Acceleration rect const	24
DEC .03125	Integ interval near earth (hours)	25
DEC .5	Integ inter in earth ref, not close (hours)	26
DEC 24.	Integration inter in sun ref hours)	27
DEC .5	Integ in T-planet ref, but not close (hours)	28
DEC .03125	Integ inter near T-planet (hours)	29
BCD 4 TEST CASE	3/8/62 GSFC TD Title	30-33
DEC 1.5	Print inter near earth (hours)	34
DEC 6.	Print inter in earth ref, not close (hours)	35
DEC 24.	Print interval in sun ref (hours)	36
DEC 24.	Print inter in T-planet ref, not close (hours)	37
DEC 24.	Print inter near T-planet (hours)	38
TRA 16382	End of data deck, non monitor	

E. Sample Input for Cartesian Coordinates - with modifications

3862	OCT 007400420742	Load radar data	7426
5075	OCT 007400421114	Compute radar output	11723
9694	OCT 002000022740	Eliminate overflow print	22736
7118	BCD IMP ORBIT WITH RADAR		
11096	DEC 4.5	Change near, far earth boundary	25530
	TRA 2,4		
	DEC 0	Cartesian coordinates	
	DEC 1	Mars is T-planet	
	DEC 102	Case number	0
	DEC 6336.4095	x coordinate km.	1
	DEC 812.60758	y coordinate km	2
	DEC 1600.4426	z coordinate km	3
	DEC .12118217	x vel component km/sec	4
	DEC 9.4967498	y vel component km/sec	5
	DEC -5.3016564	z vel component km/sec	6
	DEC 152.	Launch time in days	7
	DEC 11.	Launch time in hours	8
	DEC 3.	Launch time in minutes	9
	DEC 58.	Launch time in seconds	10
	DEC 0.	Suppress in hours	11
	DEC 1963	Year of launch	12

DEC 1	Initial ref origin indicator 1E0, 2M0, 3T0, 4S0	13
DEC 0	0 no thrust, 1 thrust, thrust indicator	14
DEC 160.	Max time of flight in hours	15
DEC 1E10	Max dist from earth in ER	16
DEC 1.	Min dist from earth in ER	17
DEC 1E10	Max dist from T-planet in ER	18
DEC 1.	Min dist from T-planet in ER	19
DEC 1E10	Max dist from sun in ER	20
DEC 1.	Min dist from sun in ER	21
DEC .02	Position rect const	22
DEC .02	Velocity rect const	23
DEC .02	Acceleration rect const	24
DEC .015625	Integ interval near earth (hours)	25
DEC .25	Integ inter in earth ref, not close (hours)	26
DEC 24.	Integration inter in sun ref (hours)	27
DEC 25.	Integ inter in T-planet ref, not close (hours)	28
DEC .015625	Integration inter near T-planet (hours)	29
BCD 4 IMP FIRST ORBIT NOMINAL		30-33

DEC .25	Print inter near earth (hours)	34
DEC 1.	Print inter in earth ref, not close (hours)	35
DEC 24.	Print interval in sun ref (hours)	36
DEC 1.	Print inter in T-planet ref, not close (hours)	37
DEC .25	Print inter near T-planet (hours)	38

F. Radar Input information

DEC 1	Number of stations	R0
BCD 4 MID ATLANTIC	station name	R1
DEC -30.,0.,0.	Long. degrees, minutes, seconds, of station	R2
DEC 5.,0.,0.	Lat. degrees, minutes, seconds, of station	R3
DEC 0.0	Altitude (feet) of station	R4
TRA 16382		

X. OUTPUT

A. Program Outputs

The following information is printed as the output of the program.

1. Title
2. Case number and any identifying titles
3. Launch time - days - hours - minutes - seconds
4. Input - in the same units as they were entered into the program
5. List of parameters used in run
6. At each rectification the following data are printed:

RECTIFICATION PRINT___(a)___REFERENCE

_(b)___PERT OVER UNPERT = ____(c)___

(a) Reference body

(b) and (c) indicate the reason for rectification

If (c) = 0 then reference body has been changed as indicated by (b) and the following nomenclature is used to indicate the new reference body:

MR - Moon Reference
SR - Sun Reference
TR - T-Planet Reference
ER - Earth Reference

If (c) $\neq$ 0 then the position, velocity, acceleration perturbations or the incremental eccentric anomaly have exceeded the permissible limits and (b) indicates which has been exceeded (See Section VI). These indications are given as:

PO - Position
VL - Velocity
AC - Acceleration
TH - Incremental Eccentric Anomaly

TIME IN DAYS, HRS, MINS, SECS, ______ ______ ______ ______ (a)

T = _________ (b)

XR _____ YR _____ ZR _____ RR _____ (c)

XRDT _____ YRDT _____ ZRDT _____ RRDT _____ (d)

RIGHT ASCENSION (DEG) = ______ DECL = ______ (e)

EARTH SUBSAT POINT LONG = ____________ (f)

LAT = ____________ (g)

HT = ____________ (h)

GHA = ____________ (i)

(a) Days, Hours, Minutes, Seconds from time of launch

(b) Print time in hours from time of launch.

(c) Position coordinates and magnitude of radius vector with respect to the reference body, kilometers

(d) Velocity components and magnitude of velocity vector with respect to the reference body, kilometers/sec

(e) Right ascension and declination in earth reference system - degrees

(f) Longitude or sub-satellite point in degrees

(g) Latitude (geodetic) in degrees

(h) The geodetic height above the earth surface in kilometers

(i) Greenwich hour angle in degrees

MOON SUBSAT POINT

LONG = ____________ (a)

LAT = ____________ (b)

AZIMUTH = ____________ (c)

ELEVATION = ____________ (d)

OSCULATING ELEMENTS AT TIME T = ____________

TRUE ANOMALY = ____________ (e)

SEM MAJ AXIS = ____________ (f)

ECCENT = ____________ (g)

PERICENT = ____________ (h)

APOCENT = ____________ (i)

INCLINATION = ____________ (j)

(a) Moon latitude – The angle between the projection of the vector from the moon to the vehicle onto the moon's orbital plane and the **moon-earth** vector (moon reference only – deg).

(b) Moon latitude – The angle between the radius vector connecting the moon and the vehicle and its projection onto the orbital plane of the moon about the earth – degrees (moon reference only).

(c) Selenocentric flight path azimuth (degrees).

(d) Selenocentric flight path angle (degrees)

(e) True anomaly (degrees)

(f) Semi-major axis of trajectory – in ER
+ = ellipse
- = hyperbola

(g) Eccentricity of trajectory**

(h) The closest distance to the reference body (not necessarily the earth) km **

(i) The farthest distance from the reference body (not necessarily the earth) km ** (meaningful only for elliptic orbits)

(j) The inclination of the orbital plane defined as the angle between the positive polar axis and the angular momentum vector. ** (Deg).

** These are the osculating values and hence only constitute an estimate of the quantities described.

ARG PERIC	=	____________	(a)
PERIOD	=	____________	(b)
MEAN MOT	=	____________	(c)
R A ASC NODE	=	____________	(d)
M ANOMALY	=	____________	(e)
E ANOMALY	=	____________	(f)
T PERIC	=	____________	(g)

UNIT PERICENTER POSITION VECTOR = (h)

UNIT ANGULAR MOMENTUM VECTOR = (i)

(a) The argument of pericenter — The angle measured from the ascending node to the pericenter vector** (DEG).
Set to zero for circular orbits and poorly determined for near circular orbits

(b) The period in hours.**

(c) The mean motion in radians per hour.**

(d) Right ascension of the ascending node measured from the vernal equinox eastward along the equator. (degrees).**

(e) Mean anomaly - radians.**

(f) Eccentric anomaly - radians.**

(g) Time of nearest pericenter — hours.**

(h) Components of the unit vector directed from reference toward pericenter.**

(i) Components of the unit angular momentum vector

** Osculating values

B. Optional Outputs

XVE = ____	YVE = ____	ZVE = ____	RVE = ____	(a)
XVM = ____	VYM = ____	ZVM = ____	RVM = ____	(b)
XME = ____	YME = ____	ZME = ____	RME = ____	(c)
XVS = ____	YVS = ____	ZVS = ____	RVS = ____	(d)
XVT = ____	YVT = ____	ZVT = ____	RVT = ____	(e)
XI = ____	ETA = ____	ZETA = ____	PERT = ____	(f)
XIDT = ____	ETADT = ____	ZETADT = ____	VPERT = ____	(g)
D2XI = ____	D2ETA = ____	D2ZETA = ____	APERT = ____	(h)

The above optional output appears between XRDT and right ascension in the standard output. For instructions on how to obtain, see Section VIII-O.

(a) Coordinates of vehicle with respect to the earth (km).

(b) Coordinates of vehicle with respect to the moon (km).

(c) Coordinates of moon with respect to the earth (km).

(d) Coordinates of vehicle with respect to the sun (km).

(e) Coordinates of vehicle with respect to the target (km).

(f) Perturbation vector and magnitude of the perturbations with respect to the reference body (km).

(g) Perturbation velocity vector and magnitude (km/sec).

(h) Perturbation acceleration vector and magnitude (km/sec^2).

C. Moon rotating and Fixed Coordinate System

XINJ = ____ YINJ = ____ ZINJ = ____ RINJ = ____ (a)

XROT = ____ YROT = ____ ZROT = ____ RROT = ____ (b)

XEM = ____ YEM = ____ ZEM = ____ REM = ____ (c)

(a) Coordinate of the vehicle wrt to a fixed coordinate system defined at injection (See Appendix Q.) (km)

(b) Coordinates of vehicle wrt to a rotating coordinate system (See Appendix Q.) (km)

(c) Coordinates of the earth wrt the moon (km).

D. Moon Osculating Elements

REFERENCE PLANE IS MOON ORBITAL PLANE
AT TIME = ________ (a)

INCLINATION	=	________	(b)
ASC NODE	=	________	(c)
ARG PERICENT	=	________	(d)
CHNG ASC NODE	=	________	(e)
INCL MOON PLANE	=	________	(f)
RA ASC NODE OF MOON	=	________	(g)
ARG OF LAT OF MOON	=	________	(h)

(a) Time in hours from launch for which the reference system is chosen. The reference plane is the osculating orbital plane of the moon. The x-axis is along the moon-earth vector. The reference system is variable or fixed, depending on the option selected (See Section VIII-Q).

(b) Inclination of vehicle plane wrt moon orbital plane (deg).

(c) Ascending node - angle between the nodal line and x-axis (deg).

(d) Argument of pericenter (deg).

(e) Change in ascending node from time zero (deg).

(f) Inclination of the moon orbital plane wrt earth's equatorial plane (deg).

(g) Right ascension of the ascending node of the moon (deg).

(h) Argument of latitude of the moon - the angle between the moon's position vector and the ascending node of the moon (deg).

E. Angle From Ascending Node to Vehicle

ANGLE FROM ASCEND NODE TO SAT = ______________ (a)

(a) Angle from the ascending node to the satellite (deg).

F. Geocentric Coordinates of Perigee

GEOCENTRIC COORDS OF PERIGEE	=	__________	(a)
LAT	=	__________	(b)
LONG	=	__________	(c)
VELOCITY	=	__________	(d)
PERICYNT VELOCITY	=	__________	(e)

(a) Print occurs in earth reference only

(b) Declination of osculating perigee point (deg).

(c) Longitude of osculating perigee point (deg).

(d) Velocity - Speed at osculating perigee point (km/sec).

(e) Pericynthion velocity - Speed at osculating pericynthion point (print occurs in moon reference only).

G. Error Print

ERROR IN LOC ____________ (a)

(a) Print octal location subsequent to where an error occurs.

H. Floating Point Spill Print

OVERFLOW IN LOC ______________ (a)

(a) Print octal location of the instruction which caused a floating point spill to occur.

I. End Print

Four blocks of storage that are useful in checking and debugging.

J. Shadow Print

$$\text{PASSAGE FROM} \left\{ \begin{array}{l} \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \\ \text{PENUMBRA} \end{array} \right\} \text{TO} \left\{ \begin{array}{l} \text{PENUMBRA} \\ \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \end{array} \right\}$$

$$\text{AT } \underline{\text{(a)}} \text{ TIME IN} \left\{ \begin{array}{l} \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \\ \text{PENUMBRA} \end{array} \right\} \underline{\text{(b)}} \text{ ACCUMULATED TIME } \underline{\text{(c)}}$$

(a) Time at which vehicle traverses denoted shadow boundary (hrs).

(b) Total time the vehicle spends in denoted shadow region during current traverse (hrs).

(c) Total accumulated time spent in denoted shadow region since launch (hrs).

K. Radar Output

STATION	(a)
AZIMUTH	(b)
ELEVATION	(b)
TOPOC. R A	(c)
TOPOC. DECL.	(c)
SLT RNG	(d)
RANGE	(e)

(a) Station name (identification) for each station

(b) Azimuth and elevation with respect to each station (deg).

(c) Topocentric right ascension and declination in degrees with respect to each station (deg).

(d) The slant range to each station (km).

(e) Rate of change of slant range for each station (km/sec).

If the elevation is negative (the vehicle is below the horizon) this print is suppressed for the station in question.

L. Flight Path Azimuth and Angle Output

GEOCENTRIC AZIMUTH = ________ (a)

ELEVATION = ________ (b)

GEOD AZIM = ________ (c)

ELEV = ________ (d)

(a) Geocentric flight path azimuth (deg).

(b) Geocentric flight path angle (deg).

(c) Geodetic flight path azimuth (deg).

(d) Geodetic flight path angle (deg).

M. Ecliptic Coordinate Output

EXES = ___	EYES = ___	EZES = ___	ERES = (a)	ELONG = (b)
EXTS = ___	EYTS = ___	EZTS = ___	ERTS = (c)	TLONG = (d)
EXVS = ___	EYVS = ___	EZVS = ___	ERVS = (e)	VLONG = (f)
XECL = ___	YECL = ___	ZECL = ___	RECL = (g)	
DXECL = ___	DYECL = ___	DZECL = ___	DRECL = (h)	

(a) Ecliptic coordinates of earth with respect to sun (km).

(b) Ecliptic longitude of earth (deg).

(c) Ecliptic coordinates of T-planet with respect to sun (km).

(d) Ecliptic longitude of T-planet (deg).

(e) Ecliptic coordinates of vehicle with respect to sun (km).

(f) Ecliptic longitude of vehicle (deg).

(g) Ecliptic coordinates of vehicle with respect to reference body (km).

(h) Ecliptic velocity of vehicle with respect to reference body (km/sec).

M. Ecliptic Coordinate Output (cont)

ANGLE BETWEEN RVE-RVS = ___(a)___ ___(b)___

ELEMENTS REFERRED TO ECLIPTIC PLANE
AT TIME T = __________

ECC	=	__________	(c)
INCL	=	__________	(d)
ASCN	=	__________	(e)
ARPG	=	__________	(f)
LONG RREF	=	__________	(g)
LAT RREF	=	__________	(h)
AZIMUTH	=	__________	(i)
ELEV	=	__________	(j)

(a) Angle between vehicle-earth and vehicle-sun vectors (deg).

(b) Angle between vehicle-T-planet and vehicle-sun vectors (deg).

(c) Eccentricity of trajectory

(d) Inclination of vehicle plane wrt ecliptic plane (deg).

(e) Ascending node - Angle between nodal line and x-axis (deg).

(f) Argument of perigee (deg).

(g) Ecliptic longitude of vehicle wrt reference body (deg).

(h) Ecliptic latitude of vehicle wrt reference body (deg).

(i) Flight path azimuth wrt (Reference body) on celestial sphere (deg).

(j) Flight path angle wrt (Reference body) on celestial sphere (deg).

N. Reentry Output

REENTRY PRINT TIME INERTIAL SPEED (km/sec)

Right ascension, declination, earth subsatellite points and flight path azimuth and angle as given above.

O. Trajectory Search Output

The output consists of the normal ITEM output for a nominal trajectory and the same trajectory output for each variation requested for each iteration. The output format used only for the trajectory search follows:

VARIATION IN INITIAL
CONDITIONS (a) (b) (c) (d) (e) (f) (g)

QUANTITY CODE (h)

DESIRED VALUES OF ABOVE QUANTITIES (i)

REQUIRED ACCURACY (j)

(a) Change in latitude (deg).

(b) Change in longitude (deg).

(c) Change in altitude (km).

(d) Change in velocity (km/sec).

(e) Change in azimuth (deg).

(f) Change in flight path angle (deg).

(g) Change in initial time (hrs).

(h) Code indicating quantities (up to 7) to be searched for.

(i) Desired values of above quantities (deg. km. sec).

(j) Tolerances allowed on above values (deg. km. sec).

O. Trajectory Search Output (cont)

MATRIX OF PARTIAL DERIVATIVES (a)

RESIDUALS AND CHANGES IN INITIAL CONDITIONS (b) (c) (d)

(a) Matrix with the dependent variables arranged by row. The independent by column.

(b) Residuals (desired-nominal) of quantities designated by the quantity code.

(c) Change required in initial conditions.

(d) Normalized changes in initial quantities in order of the variations.

P. Impact Parameter Output

IMPACT PLANE PARAMETERS DIRECTION COSINES OF

T VECTOR (a)

R VECTOR (b)

ASYMPTOTE (c)

MISS PARAMETERS CROSS T (d)

B·T, B·R (e) (f)

(a) Unit vector parallel to the ecliptic (or moon orbital plane) and perpendicular to the incoming asymptote.

(b) Unit vector perpendicular to T vector and the asymptote in a right-hand sense.

(c) Unit vector in the direction of the incoming asymptote.

(d) Time at which the impact plane is crossed (hrs).

(e) The dot product of R_{VT} at crossing time and T vector (km).

(f) The dot product of R_{VT} at crossing time and R vector (km).

(a), (b) and (c) are printed after the set up of the impact plane coordinate system. This occurs when the nominal trajectory reaches the required approach distance. This will not be repeated for subsequent trajectories unless NOMTR is reset to zero.

(d), (e) and (f) are printed after the spacecraft crosses the impact plane.

Q. Overflow Diagnostic

ADDRESS PART OF 3 PRECEDING INSTRUCTIONS
AND IR STATUS (a)

(b)

NUMBERS IN LISTED LOCATIONS,
AC, MQ (c) (d) (e)

(a) Address part of the three instructions preceding the location of the overflow.

(b) The contents of index registers 1, 2, 4 at the time of the overflow.

(c) The three numbers in the locations referred to in (a).

(d) The contents of the accumulator.

(e) The contents of the multiplier quotient register.

R. <u>Earth Fixed Velocity</u>

EARTH FIXED VELOCITY X = <u>(a)</u> Y = <u>(b)</u> Z = <u>(c)</u>

V = <u>(d)</u> AZ = <u>(e)</u> EL = <u>(f)</u>

(a) x component of the earth fixed velocity

(b) y component of the earth fixed velocity

(c) z component of the earth fixed velocity

(d) Magnitude of earth fixed velocity

(e) Geocentric flight path azimuth

(f) Geocentric flight path angle

XI. INTERNAL PROCEDURES

A. Units

The units which are used internally are earth radii and earth radii per hour in earth and moon references and astronomical units and astronomical units per hour in sun and T-planet reference system.

B. Ephemeris Tape

The relative positions of the solar system bodies are obtained from cards furnished by the U. S. Naval Observatory. A separate program prepares a binary tape referred to the mean equinox of date with 15 days per record in a form compatible with the main program. The main program searches the tape and reads in the proper file and record, keeping 30 days of tables in core storage at a time.

The first record on each file consists of the year in fixed decimal. Each of the following records contain the following information:

Word 1: Initial time of record in hours from base time.
Then 12 consecutive 15 word blocks containing the equatorial coordinates of:

XSE	YSE	ZSE	Sun wrt earth
XJS	YJS	ZJS	Jupiter wrt sun
XAS	YAS	ZAS	Mars wrt sun
XVS	YVS	ZVS	Venus wrt sun

Then three-30 word blocks containing:

XME	YME	ZME	Moon wrt earth

The moon coordinates are stored in half-day intervals ($0.0^h, 12^h.0$ UT) unit of distance is the radius of the earth. All other tables are in daily intervals ($0^h.0$ UT) the unit of distance being the AU.

The T-planet is selected by input. Jupiter is referred to as P1.

The non T-planet is designated P2.

At present, an ephemeris tape is available for 1961-1970, written in nine, two-year files. The files overlap one year.

C. Ephemeris in Core

The astronomical tables are stored in core in 24 hour intervals for the sun and the planets and 12 hour intervals for the moon. There are always 30 days of tables available arranged in such a way that the value of time for which the interpolation takes place is not near either end of the table. In earth reference, the sequence of coordinates in the tables all referred to the earth as origin is as follows:

In location "Table" there is the time of the first entry from the initial time. In "Table + 1" to "Table + 30" there are 30 x coordinates of the sun.

In "Table + 31" to "Table + 60", the y coordinates of the sun.

In "Table + 61" to "Table + 90", z coordinates of the sun.

In "Table + 91" to "Table + 180", the x, y and z coordinates of jupiter.

In "Table + 181" to "Table + 270", the x, y, z coordinates of mars or venus, whichever is not T-planet.

In "Table + 271" to "Table + 360", the x, y, z coordinates of T-planet.

In "Table + 361" to "Table + 420", the x coordinates of the moon.

In "Table +421" to "Table + 480", the y coordinates of the moon and in "Table + 481" to "Table +540", z coordinates of the moon.

Table 2 shows the arrangement of "Tables" for other reference systems

TABLE 2 - ASTRONOMICAL TABLES IN CORE

Chart 1 shows the location of the first x coordinates of the various attracting bodies in other references. The beginning of the corresponding y and z tables is found by adding 30 and 60 respectively to these locations for the non moon tables, and 60 and 120 for the moon tables.

	Table +1	Table +91	Table +181	Table +271	Table +361
earth ref	XSE,	XP1E,	XP2E,	XTE,	XME
moon ref	XSM,	XP1M,	XP2M,	XTM,	XEM
sun ref	XES,	XP1S,	XP2S,	XTS,	XME
T-planet ref	XST,	XP1T,	XP2T,	XET,	XME

In sun and T-planet reference the perturbation of the moon is not important. The moon table is spaced at 24 hour intervals. These tables are in XMOT to ZMOT + 29. These locations contain XMS, YMS and ZMS in sun reference and XMT, YMT and ZMT in T-planet reference respectively.

D. Perturbation Program

The perturbation program solves three differential equations for XI, ETA, ZETA. The differential equation for XI, with the various terms replaced by the storages containing them, is representative of all three equations and is given on the next page.

$$
\begin{aligned}
D2XI = &- GME\ (VCOR + 0/VCOR + 3 \quad -COMP + 0/COMP + 3)\\
&- GMM\ (VCOR + 18/VCOR + 21 \quad -COMP + 18/COMP + 21)\\
&- GMS\ (VCOR + 12/VCOR + 15 \quad -COMP + 12/COMP + 15)\\
&- GMP1\ (VCOR + 24/VCOR + 27 \quad -COMP + 24/COMP + 27)\\
&- GMP2\ (VCOR + 30/VCOR + 33 \quad -COMP + 30/COMP + 33)\\
&- GMT\ (VCOR + 6/VCOR + 9 \quad -COMP + 6/COMP + 9)\\
&+ OTHER\ PERTURBATIONS
\end{aligned}
$$

where, e.g., in the first term GME - K^2 (mass of earth), and VCOR + 3 is the length cubed of the vector (VCOR + 0, VCOR + 1, VCOR + 2). Similarly, in the other terms the demoninator is the length cubed of the corresponding numerator. In case the two terms within each parentheses are nearly equal, they are computed by the special method described in Appendix E to avoid loss of accuracy. The contents of the COMP storage at any time t depends on the reference origin at that time. For details see Tables 3 and 4.

TABLE 3 - CONTENTS OF COMP TO COMP + 33

EARTH REF	MOON REF	SUN REF	T-PLANET REF	STORAGE
XV0	XME	XSE	XTE	COMP + 0, + 1, + 2,3,4,5 R^3,R,R^2
XET	XMT	XST	XVT0	COMP + 6, + 7, + 8
XES	XMS	XVS0	XTS	COMP +12, +13, +14
XEM	XVM0	XSM	XTM	COMP +18, +19, +20
XEP1	XMP1	XSP1	XTP1	COMP +24, +25, +26
XEP2	XMP2	XSP2	XTP2	COMP +30, +31, +32

Here XVE refers to the x component of the vector vehicle wrt earth, with corresponding definitions for the other quantities. An additional subscript of 0 denotes quantity derived from the two-body problem.

TABLE 4 - CONTENTS OF VCOR TO VCOR +33

ALL REFERENCES	STORAGE
XVE	VCOR + 0, + 1, + 2
XVT	VCOR + 6, + 7, + 8
XVS	VCOR +12, +13, +14
XVM	VCOR +18, +19, +20
XVP1	VCOR +24, +25, +26
XVP2	VCOR +30, +31, +32

The instantaneous reference origin is shown by the sense lights on the console as follows:

Sense light 1 "on" is earth or moon reference.
(0 in the location LUNRF is earth reference, 1 in location LUNRF is moon reference)
Sense light 3 "on" is T-planet reference.
Sense light 4 "on" is sun reference.

XII. OPERATING PROCEDURES

A. Introduction

The following procedures for running program are assembled to serve as a guide for programmers and IBM 7090 machine operators when using the Interplanetary Trajectory Encke Method program. It is strongly advised that all input should be listed and examined carefully before using program and fastened to output when received, to provide a permanent record of input for case run.

B. Normal Operation (Non Monitor)

1. Input: The data are supplied on cards. They should be loaded on tape and mounted on A2. The last card of the input data should be a TRA 16382 (for non monitor operation) beginning in column 8. This card will cause an end of file to be written on the output tape, after completion of all cases. The normal program stop in this case will show location 37777 in the instruction counter and HTR 37777 in storage.

2. Output: The output of the program is on tape A3.

3. Ephemeris Tapes: A variety of ephemeris tapes (tables of planetary positions as functions of time and year) may be used. Normally a 10 year tape will be used. The particular tape desired will be noted with the input instructions by tape number. These tapes may be either high or low density and should be so marked on the tape reel along with the other identifying information. The ephemeris tape should be mounted on A5 with the appropriate density. These tapes should all be FILE PROTECTED.

4. Program Tape: This tape will normally be specified by the programmer (FILE PROTECTED). It has two files, one for the ITEM program, and one used to restart the program after it has been "interrupted" (see XII-D). This tape is placed on B5.

5. Call Card: The call card is used to call in a particular file on B5. The address portion of the keys on the console indicate which file is desired (e.g. 35 down, indicates first file, 34 second, etc.)

C. Monitor Operation

For monitor operation the card TRA 16382 is omitted and the program followed by the input is loaded off line on A2. The end of file mark on A2 after all cards are read will then return control to the monitor. The loader (first card) has to be replaced by a A2 loader.

The instructions on the call card are as follows:

00000	0	00025	0	00003	IOCD	
00001	0	06000	0	00001	TCOA	
00002	0	76000	0	00004	ENK	
00003	0	13100	0	00000	XCA	
00004	0	73400	1	00000	PAX	
00005	0	77200	0	02205	REWB	5
00006	-2	00001	1	00014	TNX	14, 1, 1
00007	0	76200	0	02225	RTBB	5
00010	-0	54000	0	00022	RCHB	
00011	0	06100	0	00011	TCOB	
00012	2	00001	1	00007	TIX	
00013	-0	03000	0	00014	TEFB	
00014	-0	02200	0	00015	TRCB	
00015	0	76200	0	02225	RTBB	5
00016	-0	54000	0	00021	RCHB	
00017	-0	54400	0	00000	LCHB	
00020	0	02000	0	00001	TRA	
00021	-1	00003	0	00000	IOCT	
00022	2	00000	0	00000		
00023	1	00000	0	00022		

1. Operating Procedure:

   a. Place input tape on A2
   b. Place output tape on A3
   c. Place Ephemeris tape on A5
   d. Place program tape on B5
   e. All sense switches up
   f. Key number 35 down (for first file on B5)
   g. Clear machine
   h. Ready B5 call card in card reader
   i. Press load cards key

Program will start and continue to normal stop. IC 37777 and HTR 37777 in storage.

D. Interrupt Features

There are two interrupt features operated by sense switches and designed for use with (a) either many short cases or (b) a few long cases. Pressing down sense switch 4 prints the current and the maximum time on line. By knowing the length of time the program has been on the machine the time remaining to completion can be estimated by direct proportionality. From this, one can decide on the advisability of using these optional interrupt features. The first option is used with short cases permitting the program to complete the case it is working on. The second option will stop a case immediately and dump core onto B4 so the case may be resumed at a later time.

1. Interrupt Operations: no restart

   a. Place sense switch 5 down
   b. Program will stop at end of case, HTR 37777

2. Interrupt Operations: with restart

   a. Ready B4
   b. Place sense switch 6 down
   c. Two stops are possible, HTR 37777 - normal stop, HTR 12647 - several failures to write good B4, probably due to bad tape unit. See step e.

d. After normal stop all tapes may be removed, unless the input tape A2 contains additional cases which are to be run. In that case A2 should not be touched.
e. At CSPO-1 HTR 12647 (STBLK-1) the B4 tape unit should be changed, then follow next instructions.
f. Place sense switch 6 up
g. Press start
h. Place sense switch 6 down
i. See step c.

E. The procedures outlined in this manual are the normal functions of the program and are available for flexibility in operation. Departures from these procedures or other program stops should be called to the attention of the programmer before taking problem off the machine.

F. Restart Feature:

If sense switch 6 has been used and the program written on B4 there are two methods of restarting the program.

Method 1

1. Ready the previously written B4 on tape unit B4
2. Follow the procedure outlined in Section XII-B under "Normal Operations" except for step 5.-f.
3. For step 5.-f. - place key number 34 down instead of number 35. This calls second file (i.e. restart program) on B5.

Method 2

1. Ready B4
2. Ready other tapes as in Section XII-B under "Normal Operations"
3. All sense switches up
4. Clear machine
5. Ready 8 card "Restart" program in card reader
6. Press load cards key

Program will restart and continue to normal stop. IC 37777 and HTR 37777.

G. Alternate method of operation:

1. Occasionally it is useful to place the data immediately behind the binary deck and load the program directly into the machine from tape A2. The A2 loader must replace the B5 loader as the first card on the deck and the A2 call card used to initiate the program.

2. To run with deck loaded on A2

   a. Place A2 loader on binary deck
   b. Place data behind binary deck
   c. Load cards onto tape, place tape on A2
   d. Place planetary ephemeris tape on A5
   e. Key 35 down
   f. Place A2 call card in card reader
   g. Press load cards key
   h. Output is on A3

3. Error Stops

   Numerous error stops are provided when the machine detects inconsistencies. In this case, the machine prints "ERROR AT LOCATION.....", then prints a number of working storages to aid in finding the cause of the error and proceeds to the next case. Most frequently, such conditions are caused by faulty input. As a first step, therefore, the input should be examined carefully. In this procedure it is extremely helpful to list the input before executing the program and clipping it to the output.

H. Machine Requirements:

The program uses a 32K, 2 channel IBM 7090 or 7094 computer. It uses 3 tapes and 1 printer on A channel and 1 tape on B channel, and a second tape on B channel if the save-restore feature is used. For monitor operation or the alternate mode described in Section XII-G. the B-channel is unnecessary.

I. Additional Information

The work on this program continues and additional features become available from time to time. Please contact F. B. Shaffer, Jr., Goddard Space Flight Center, Theoretical Division, Greenbelt, Md., for additional information.

XIII. LIST OF IMPORTANT LOCATIONS IN THE PROGRAM

This section presents a list of constants and other locations which will enable the program user to modify the computations for special purposes.

The majority of the solar system constants, e.g., earth gravitational harmonics, planetary masses etc., are found in the program listing beginning at 25415. The first group of coefficients are used in the series expansion for the sub-satellite computation. Notable exceptions are as follows:

A. In Radar

22326	Z$ESQ	9430	earth's eccentricity squared
22327	Z1MESQ	9431	one minus earth's eccentricity squared
22332	Z$F	9434	reciprocal of earth's radius in feet

B. In Drag Table

32645	DRSC	13733	negative earth radius in centimeters drag scaler

C. Other Locations are:

25604	XRDT	11140	velocity of vehicle with respect to reference body
25573	XRODT	11131	unperturbed velocity of vehicle with respect to reference body
25764	T	11252	time from launch (hrs)
25765	h=T+1	11253	current integration interval (hrs)
25766 25770	XI	11254 11256	perturbation displacement
25771 25773	XI+3	11257 11259	perturbation velocity
25774 25776	D2XI	11260 11262	perturbation acceleration
26760	TABLE	11760	beginning of a block of locations described in Section XI

REFERENCES

1. Goddard Minimum Variance Orbit Determination Program. Report No. X-640-62-191, Special Projects Branch, Theoretical Division, October 18, 1962.

2. The Structure of the High Atmosphere, C. A. Whitney, Smithsonian Institution, Astrophysical Observatory, Special Report, No. 25, April 20, 1959.

3. The ARDC Model Atmosphere, 1956, R. A. Minzner, W. S. Ripley, December, 1956. AFCRCTW-56-204, Astia Doc. 110233.

4. Comparison of Special Perturbation Methods in Celestial Mechanics, S. Pines, M. Payne, H. Wolf, August 1960, ARL TR 60-281.

5. The Reduction from Geocentric to Geodetic Coordinates, J. Morrison, S. Pines, A. J. 66, No. 1, 1961, February - No. 1286.

MATHEMATICAL APPENDIX

A. INTRODUCTION

The problem of orbit determination over long time periods requires a precise technique for integrating the equations of motion. Reference 4 contains an analysis of integration procedure that yields the minimum loss of information due to the accumulation of numerical round-off errors. The Encke perturbation method has been shown to require minimum machine computation time for a minimum loss of numerical accuracy. The orbit prediction scheme presented herein uses a modified form of the Encke method with the initial position and velocity vectors replacing the conventional P and Q vectors of the Encke scheme. By avoiding reference to the position of perigee, it is possible to avoid numerical ambiguities arising from near circular orbits and orbits of low inclination.

B. EQUATIONS OF MOTION

In a Newtonian system, the equations of motion of a particle in the gravitational field of n attracting bodies and subject to other perturbing accelerations such as thrust, drag, oblateness, radiation pressure, etc. are given by

$$\ddot{\mathbf{R}}_v = -\sum_{i=1}^{n} \mu_i \frac{\mathbf{R}_{vi}}{r_{vi}^3} + \sum_j \mathbf{F}_j \tag{B.1}$$

These equations are put into observable form by referring them to a reference body c. The equations of motion of the reference body are

$$\ddot{\mathbf{R}}_c = -\sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \frac{\mathbf{R}_{ci}}{r_{ci}^3} \tag{B.2}$$

Subtraction of Equation (B.2) from Equation (B.1) results in the equation of motion of the vehicle with respect to the reference body c.

$$\ddot{\mathbf{R}}_{vc} = -\left(\mu_v + \mu_c\right) \frac{\mathbf{R}_{vc}}{r_{vc}^3} - \sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \left[\frac{\mathbf{R}_{vi}}{r_{vi}^3} - \frac{\mathbf{R}_{ci}}{r_{ci}^3}\right] + \sum_j \mathbf{F}_j \tag{B.3}$$

C. METHOD OF INTEGRATION

If Equation (B.3) is integrated directly by some numerical scheme, there results, after a number of step-by-step integrations, an accumulation of error which leads to inaccurate results. To avoid this loss in precision, it is convenient to write Equation (B.3) in the form

$$\ddot{R}_{vc} = \ddot{R}_k + \Delta\ddot{R} \tag{C.1}$$

The velocity and displacement vectors can be written as

$$\dot{R}_{vc} = \dot{R}_k + \Delta\dot{R} \tag{C.2}$$

$$R_{vc} = R_k + \Delta R \tag{C.3}$$

The reference body is chosen so as to minimize the perturbations, i.e. the one in whose sphere of influence the vehicle travels.

In this method $\ddot{R}_k$ is taken as

$$\ddot{R}_k = -\left(\mu_v + \mu_c\right) \frac{R_k}{r_k^3} \tag{C.4}$$

and

$$\Delta\ddot{R} = -\left(\mu_v + \mu_c\right)\left[\frac{R_{vc}}{r_{vc}^3} - \frac{R_k}{r_k^3}\right] - \sum_{\substack{i=1 \\ i \neq c}}^{n} \mu_i \left[\frac{R_{vi}}{r_{vi}^3} - \frac{R_{ci}}{r_{ci}^3}\right] + \sum_j F_j \tag{C.5}$$

Equations (C.4) constitute the equations of motion of the Kepler problem and are solved as described in appendix D. Equations (C.5) are integrated numerically. The integration scheme employed by the ITEM program is a sixth order backward difference scheme, initiated by a Runge-Kutta scheme. The routine used is "RW DE6F Floating Point Cowell Second Sum integration of second order differential equations." (SHARE distribution #775.)

As derived in appendix D, the solution of the Kepler problem may be represented by the vectors R_0, $\dot{R}_0$, the scalar a and the rectification time t_0.

The rectification process consists of moving R_{vc}, $\dot{R}_{vc}$ into the locations R_0 and $\dot{R}_0$, t into t_0 and the computation of a and n.

For computational convenience the coefficients appearing in equations D6 are also computed during rectification.

D. SOLUTION OF THE KEPLER PROBLEM

The two-body orbit which results from the solution of Equation (C.4) with the initial conditions:

$$
\begin{aligned}
R_k(t_0) &= R_{vc}(t_0) = R_0 \\
\dot{R}_k(t_0) &= \dot{R}_{vc}(t_0) = \dot{R}_0
\end{aligned}
\qquad (D.1)
$$

See Figure 4.

R_k and $\dot{R}_k$ may be expressed as linear combinations of R_0 and $\dot{R}_0$.

$$
\begin{aligned}
R_k &= f R_0 + g \dot{R}_0 \\
\dot{R}_k &= \dot{f} R_0 + \dot{g} \dot{R}_0
\end{aligned}
\qquad (D.2)
$$

The functions f and g can be expressed completely in terms of the incremental eccentric anomaly $\theta = \mathbf{E} - \mathbf{E}_0$ and the terms $e \sin \mathbf{E}_0$ and $e \cos E_0$ which can be unambiguously expressed in terms of R_0 and $\dot{R}_0$.

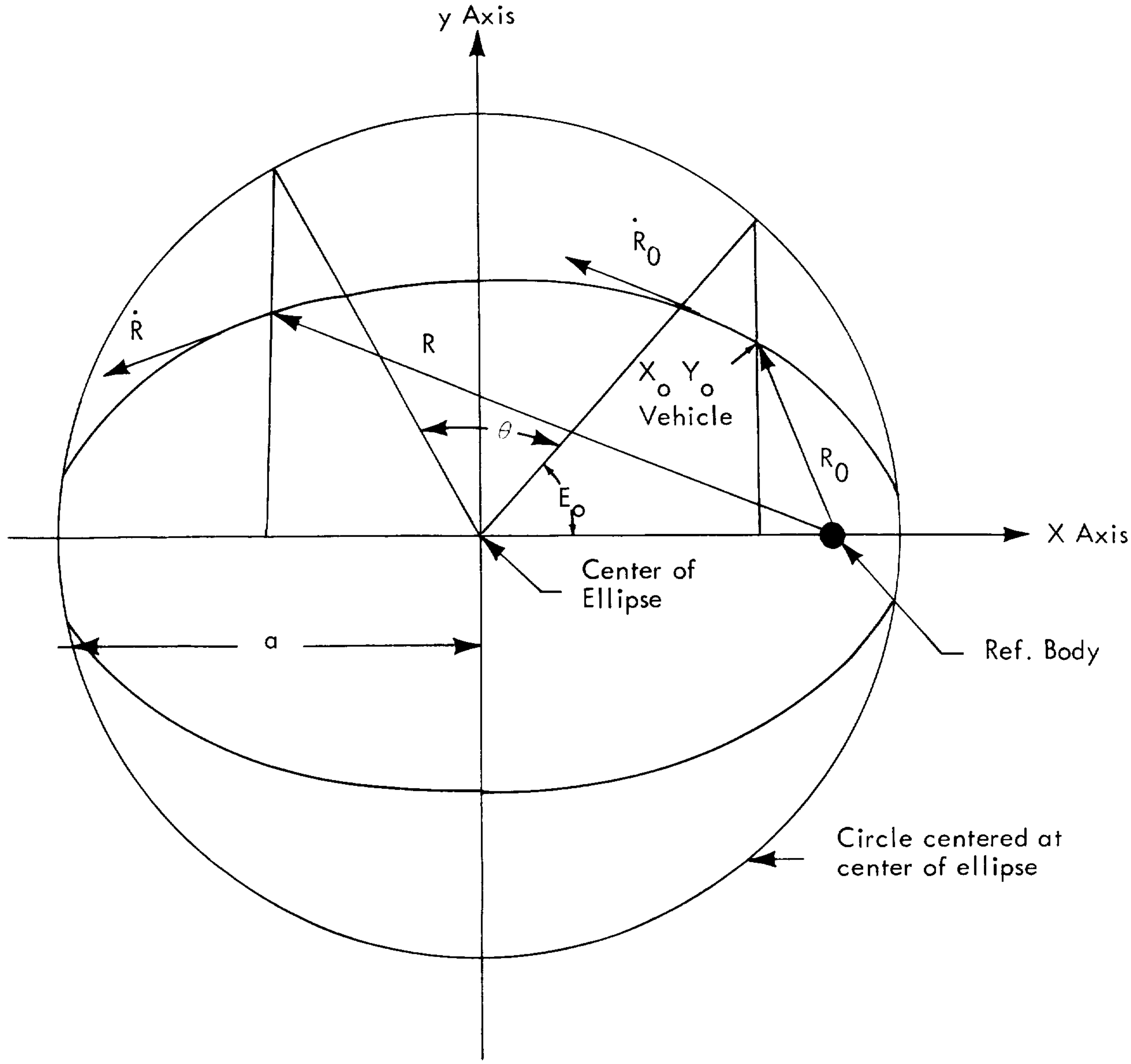


Figure 4. The Geometry of the Elliptic Kepler Orbit.

The following expressions for f and g result:

$$
\left.
\begin{aligned}
f &= -\frac{a}{r_0}\left[-\cos\theta + 1\right] + 1 \\
g &= \sqrt{\frac{a}{\mu}}\, r_0 \sin\theta + \frac{a d_0}{\mu}\left(-\cos\theta + 1\right) = \frac{\theta - \sin\theta}{-n} + t - t_0 \\
\frac{r}{a} &= \left(1 - \cos\theta\right) + \frac{r_0}{a}\cos\theta + \frac{d_0}{\sqrt{\mu a}}\sin\theta \\
\dot{f} &= -\frac{\sqrt{\mu a}}{r r_0}\sin\theta \\
\dot{g} &= -\frac{a}{r}\left[1 - \cos\theta\right] + 1
\end{aligned}
\right]
\quad \begin{matrix}\text{Elliptic} \\ \text{(D.3)}\end{matrix}
$$

where:

$$d_0 = R_0 \cdot \dot{R}_0$$

$$a = \left(\frac{2}{r_0} - \frac{v_0^2}{\mu}\right)^{-1}$$

$$r_0 = \left(R_0 \cdot R_0\right)^{1/2}$$

and

$$v_0^2 = \dot{R}_0 \cdot \dot{R}_0$$

$$
\left.\begin{aligned}
f &= \frac{a}{r_0}\left[(\cosh\theta - 1)\right] + 1 \\
g &= \sqrt{\frac{-a}{\mu}}\, r_0 \sinh\theta - \frac{ad_0}{\mu}(\cosh\theta - 1) \\
&= -\frac{\sinh\theta - \theta}{n} + t - t_0 \\
\frac{r}{a} &= -(\cosh\theta - 1) + \frac{r_0}{a}\cosh\theta - \frac{d_0}{\sqrt{-\mu a}}\sinh\theta \\
\dot{f} &= -\frac{\sqrt{-\mu a}}{r r_0}\sinh\theta \\
\dot{g} &= \frac{a}{r}(\cosh\theta - 1) + 1
\end{aligned}\right]\quad \text{Hyperbolic} \qquad (D.3)
$$

A similar technique applied to Kepler's Equations results in the following equation, which furnishes θ as a function of the time.

$$
n\left(t - t_0\right) = \theta - \sin\theta + \frac{r_0}{a}\sin\theta + \frac{d_0}{\sqrt{\mu a}}(1 - \cos\theta) \quad \text{Elliptic}
$$

$$
n\left(t - t_0\right) = \sinh\theta - \theta - \frac{r_0}{a}\sinh\theta + \frac{d_0}{\sqrt{-\mu a}}(\cosh\theta - 1) \quad \text{Hyperbolic} \qquad (D.4)
$$

If the functions f_1, f_2, f_3, f_4 are defined as

	Ellipse	Hyperbola
$f_1(\theta)$	$= \theta - \sin\theta$	$= -\theta + \sinh\theta$
$f_2(\theta)$	$= 1 - \cos\theta$	$= -1 + \cosh\theta$
$f_3(\theta)$	$= \sin\theta$	$= \sinh\theta$
$f_4(\theta)$	$= \cos\theta$	$= \cosh\theta$

(D.5)

The solution of the two-body problem for both elliptic and hyperbolic orbits is given by:

$$
\begin{aligned}
f &= \frac{-|a|}{r_0} f_2 + 1 \\
g &= -\frac{1}{n} f_1 + (t - t_0) \\
\left|\frac{r}{a}\right| &= f_2 + \frac{r_0}{|a|} f_4 + \frac{d_0}{\sqrt{\mu |a|}} f_3 = F'(\theta) \\
\dot{f} &= -\sqrt{\frac{\mu}{|a|}} \frac{1}{r_0} \frac{|a|}{r} f_3 \\
\dot{g} &= \frac{-|a|}{r} f_2 + 1 \\
n(t - t_0) &= f_1 + \frac{r_0}{|a|} f_3 + \frac{d_0}{\sqrt{\mu |a|}} f_2 = F(\theta) = \Delta M
\end{aligned}
\tag{D.6}
$$

E. NUMERICAL PROCEDURES

1. Solution of Kepler's equation

The last of equation (D.6) which is a monotonically increasing function is solved for θ by Newton's method as follows: let θ_0 = previous value of θ obtained, better values of θ are obtained by computing

$$\theta_{j+1} = \theta_j - \frac{F(\theta_j) - \Delta M}{F'(\theta_j)} \tag{E.1}$$

until convergence is attained. From θ and equation (D.6) plus equation (D.2) the position and velocity in the plane of the reference orbit are established. The series expansion for f_1 and f_2 are given below:

$$\left.\begin{aligned} f_1 &= \theta - \sin\theta = -\theta\left\{\left(\left(\left(\frac{-\theta^2}{27\cdot 26}+1\right)\frac{-\theta^2}{25\cdot 24}+1\right)\cdots+1\right)\frac{-\theta^2}{3\cdot 2}\right\} \\ f_2 &= 1 - \cos\theta = -\left\{\left(\left(\left(\frac{-\theta^2}{26\cdot 25}+1\right)\frac{-\theta^2}{24\cdot 23}+1\right)\cdots+1\right)\frac{-\theta^2}{2\cdot 1}\right\} \end{aligned}\right] \text{Elliptic}$$

$$\left.\begin{aligned} f_1 &= \sinh\theta - \theta = \theta\left\{\left(\left(\left(\frac{\theta^2}{27\cdot 26}+1\right)\frac{\theta^2}{25\cdot 24}+1\right)\cdots+1\right)\frac{\theta^2}{3\cdot 2}\right\} \\ f_2 &= \cosh\theta - 1 = \left(\left(\left(\frac{\theta^2}{26\cdot 25}+1\right)\frac{\theta^2}{24\cdot 23}+1\right)\cdots+1\right)\frac{\theta^2}{2\cdot 1} \end{aligned}\right] \text{Hyperbolic} \tag{E.2}$$

In order to insure a minimum loss of accuracy, the method of computation of f_1, f_2, f_3, f_4 will depend upon the magnitude of θ. The following tests are made to determine the method of computation of f_1, f_2, f_3, f_4.

Hyperbolic Case

(a) If $|\theta| < 1$.

f_1 and f_2 are computed by Equations (G.2)

$$f_3 = \theta + f_1$$
$$f_4 = 1 + f_2 \qquad (E.3)$$

(b) If $|\theta| > 1$.

Compute $f_0 = e^{\theta}$

$$f_3 = \frac{1}{2}\left(f_0 - \frac{1}{f_0}\right)$$

$$f_4 = \frac{1}{2}\left(f_0 + \frac{1}{f_0}\right)$$

$$f_1 = f_3 - \theta \qquad (E.4)$$

$$f_2 = f_4 - 1$$

<u>Elliptic Case</u>

(c) If $|\theta| < \frac{\pi}{3}$

f_1 and f_2 are computed by Equations (E.3)

$$f_3 = \theta - f_1$$
$$f_4 = 1 - f_2 \qquad (E.5)$$

(d) If $|\theta| > \frac{\pi}{3}$

Then f_4 is computed by means of Rand polynomials

$$f_2 = 1 - f_4$$

Also if $|\theta - \sin\theta| < |\sin\theta|$

(or approximately if $|\theta| < 1.9$ (E.6)

f_1 is computed by Equations (E.3)

$$f_3 = \theta - f_1$$

Otherwise if $|\theta - \sin\theta| > |\sin\theta|$

(or approximately if $|\theta| > 1.9$

f_3 is computed by means of Rand polynomials and

$$f_1 = \theta - f_3$$

2. Special Computation of Perturbation terms

A special problem arises in the computation of the terms involved in Equations (C.5) due to the loss of accuracy in subtracting the nearly equal terms involved. An expression based on the binomial expansion removes this difficulty. This method supplies results more accurate than the straight-forward computation for terms of the type of:

$$\frac{R}{r^3} - \frac{R_0}{r_0^3}$$

if $|R - R_0|$ is small compared to r and is known more accurately then can be computed by taking the difference between R and R_0.

This is the case, for instance, for the Encke term

$$\frac{R_{VC}}{r_{VC}^3} - \frac{R_k}{r_k^3}$$

since R_{VC} is computed from

$$R_{VC} = R_k + \Delta R$$

and ΔR is small and known more accurately than can be computed from $R_{VC} - R_k$.

Another example will be the sun's and planets' perturbation in earth reference and the moon's perturbations for a satellite near the earth. For example

$$R_{VS} = R_{ES} + R_{VE}$$

R_{VE} is small and known more accurately than can be computed from $R_{VS} - R_{ES}$.

$$R = R_0 + \Delta R$$

$$\frac{R}{r^3} - \frac{R_0}{r_0^3} = \frac{R}{r^3} - \frac{R}{r_0^3} + \frac{R}{r_0^3} - \frac{R_0}{r_0^3}$$

$$= \frac{R}{r^3}\left[1 - \left(\frac{r}{r_0}\right)^3\right] + \frac{\Delta R}{r_0^3}$$

$$= \frac{\Delta R}{r_0^3} + \frac{R}{r^3}\left[1 - (1 - u)^{3/2}\right] \qquad \text{(E.7)}$$

or finally:

$$\frac{R}{r^3} - \frac{R_0}{r_0^3} = \frac{\Delta R}{r_0^3} + \frac{R}{r^3} \sum_{n=1}^{6} a_n u^n \tag{E.8}$$

where

$$u = \frac{2}{r_0^2}\left(R_0 + \frac{1}{2}\Delta R\right) \cdot \Delta R$$

$$a_1 = -\frac{3}{2}, \quad a_2 = -\frac{3}{8}, \quad a_3 = \frac{1}{16}, \quad a_4 = -\frac{3}{128}, \tag{E.9}$$

$$a_5 = \frac{3}{256}, \quad a_6 = \frac{-7}{1024}$$

The six terms are adequate for $|u| < 0.1$. For larger values of u straightforward computation is adequate.

F. CONCLUSIONS

The method presented yields accurate trajectories using relatively little computer time. Summarizing some of the important features:

1. All significant solar-system bodies may be included without undue complications.

2. Since the perturbations only are integrated, the allowable integration interval is fairly large over most of the path. Even in the vicinity of earth and target a relatively large interval (compared to other schemes) may be used without limiting the stability and accuracy of the solutions.

3. The perturbations are kept small in two ways. First, the two-body orbit is rectified whenever the perturbations exceed a specified maximum value compared to the corresponding unperturbed values. This limits error build-up with respect to a particular reference body. Second, the reference body of the two-body problem is changed from earth, to sun, to target, etc., according as that body would otherwise contribute the largest perturbing force.

4. The advantages of the NASA two-body equations are these: First of all, in deriving the equations there are many algebraic cancellations which occur and result in greater accuracy. In the classical solution a determination of P and Q was necessary. A numerical error was introduced by the process of rectification. For the NASA two-body formulation only a transfer of R and $\dot{R}$ into the locations R_0 and $\dot{R}_0$ is necessary for rectification.

Second, these equations will handle circular orbits, zero inclination etc. Third, the problem is defined in terms of parameters which have real physical significance, (namely, the position and velocity vector) which are directly relatable to measurable quantities.

G. OBLATENESS TERMS

The oblateness perturbation terms in equations (C.5) are derived from the potential given by equation (G.1):

$$\Phi = \frac{\mu}{r}\left\{-\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right] - \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{5}{2}\left(\frac{z}{r}\right)^3 - \frac{3}{2}\left(\frac{z}{r}\right)\right] - \left(\frac{a_e}{r}\right)^4 J_{40}\left[\frac{35}{8}\left(\frac{z}{r}\right)^4 - \frac{15}{4}\left(\frac{z}{r}\right)^2 + \frac{3}{8}\right]\right\} \tag{G.1}$$

where a_e = equatorial radius of the earth

$$\mathbf{F} = \text{grad}\,\Phi$$

This vector can be written in the form

$$\mathbf{F} = \ell\mathbf{R} - m\hat{k}$$

where $\hat{k}$ is a unit vector in the z - direction.

$$\ell = \frac{\mu}{r^3}\left\{\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 - \frac{3}{2}\right] - \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{35}{2}\left(\frac{z}{r}\right)^3 - \frac{15}{2}\left(\frac{z}{r}\right)\right] - \left(\frac{a_e}{r}\right)^4 J_{40}\left[\frac{315}{8}\left(\frac{z}{r}\right)^4 - \frac{105}{4}\left(\frac{z}{r}\right)^2 + \frac{15}{8}\right]\right\} \tag{G.2}$$

$$m \quad \frac{\mu}{r^2}\left\{\left(\frac{a_e}{r}\right)^2 J_{20} \left(3\frac{z}{r}\right) \cdot \left(\frac{a_e}{r}\right)^3 J_{30}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 \cdot \frac{3}{2}\right]\right. \tag{G.3}$$

$$\left. \cdot \left(\frac{a_e}{R}\right)^4 J_{40}\left[\frac{35}{2}\left(\frac{z}{r}\right)^3 - \frac{15}{2}\frac{z}{r}\right]\right\}$$

The equatorial oblateness is introduced by the potential:

$$/_{EQ} \quad \frac{\mu}{r}\left\{\left(\frac{a_e}{r}\right)^2 J_{EQ}\left[\frac{3}{2}\left(\frac{y}{r}\right)^2 \quad \frac{1}{2}\right]\right\} \tag{G.4}$$

$$F_{EQ} \quad /_{EQ}\ R \cdot m_{EQ}\ \hat{j}'$$

Where y' is the distance from earth-fixed meridian plane containing the longest equatorial radius and $\hat{j}'$ is a unit vector normal to this plane.

$$/_{EQ} \quad \frac{\mu}{r^3}\left\{\left(\frac{a_e}{r}\right)^2 J_{EQ}\left[\frac{15}{2}\left(\frac{y'}{r}\right)^2 \cdot \frac{3}{2}\right]\right\} \tag{G.5}$$

$$m_{EQ} \quad -\frac{\mu}{r^2}\left\{\left(\frac{a_e}{r}\right)^2 J_{EQ}\left(3\frac{y'}{r}\right)\right\}$$

The expressions obtained are equivalent to the usual method of expressing the force in terms of latitude and longitude.

H. TRANSFORMATION EQUATIONS FROM GEODETIC POLAR COORDINATES TO CARTESIAN COORDINATES*

The geodetic polar coordinates in the program are referred to an ellipsoid of revolution. The equation of a cross section is given by

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \qquad \text{(H.1)}$$

where

$$b^2 = a^2\left(1 - e^2\right)$$

The slope of the normal, along which h is measured is given by

$$\tan\phi = -\frac{1}{\frac{dz}{dx}} = \frac{a^2 z}{b^2 x} \qquad \text{(See Figure 5)} \qquad \text{(H.2)}$$

and

$$\tan\phi' = \frac{z}{x} = \frac{b^2}{a^2}\tan\phi = \left(1 - e^2\right)\tan\phi$$

Eliminating x between equations (H.1) and (H.2) and solving for z results in:

$$z = \frac{a\left(1 - e^2\right)\sin\phi}{\left(1 - e^2\sin^2\phi\right)^{1/2}}$$

*For geocentric (i.e. $e^2 = 0$) polar coordinates, c = s = 1. In this case the latitude input is interpreted as declination.

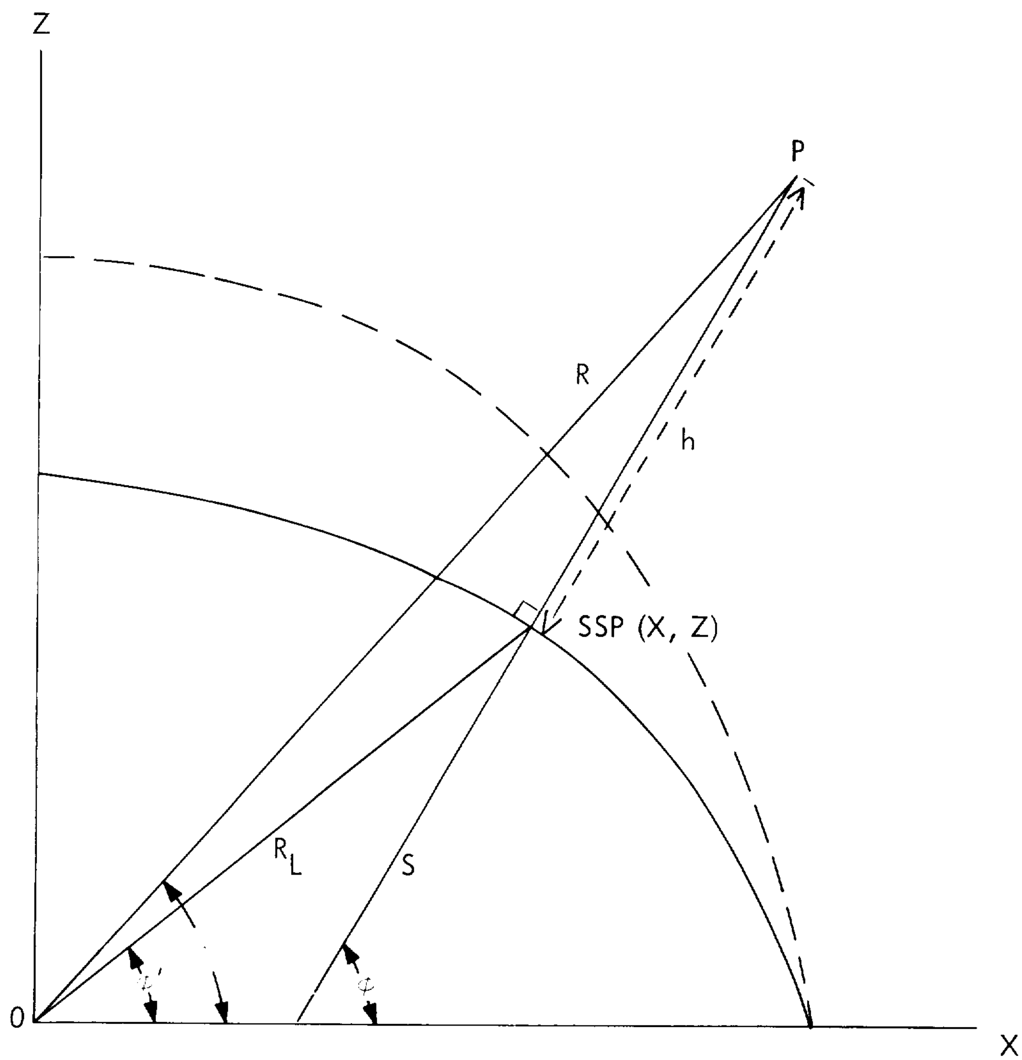


Figure 5. Relation Between Declination, Geocentric and Geodetic Latitudes

and from equation (H.2) then

$$x = \frac{a \cos \phi}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}}$$

In units of a_e, R and $\dot{R}$ are then given by equation (H.3)

$$
\begin{aligned}
c &= \left(1 - e^2 \sin^2 \phi\right)^{-1/2} \\
s &= \left(1 - e^2\right) c \\
x &= (c + h) \cos \phi \cos \left(\theta - \theta_0\right) \\
y &= (c + h) \cos \phi \sin \left(\theta - \theta_0\right) \\
z &= (s + h) \sin \phi \\
\dot{x} &= v \left\{ (\sin \gamma \cos \phi - \cos \gamma \cos A \sin \phi) \cos \left(\theta - \theta_0\right) \right. \\
&\qquad \left. - \cos \gamma \sin A \sin \left(\theta - \theta_0\right) \right\} \\
\dot{y} &= v \left\{ (\sin \gamma \cos \phi - \cos \gamma \cos A \sin \phi) \sin \left(\theta - \theta_0\right) \right. \\
&\qquad \left. + \cos \gamma \sin A \cos \left(\theta - \theta_0\right) \right\} \\
\dot{z} &= v \left\{ \sin \gamma \sin \phi + \cos \gamma \cos A \cos \phi \right\}
\end{aligned}
\tag{H.3}
$$

These equations include the effect of the rotation of the earth. The longitude of the vernal equinox $\left(\theta_0\right)$ at launch time is computed by the program from Newcomb's formula.

I. TRANSFORMATION EQUATIONS FOR RADAR SIMULATION

The program computes sight angles (in an azimuth-elevation system), slant range and range rate data for up to 30 radar stations. The vehicle coordinates are transformed from a system of geocentric cartesian coordinates (xyz), the x-axis in the direction of the vernal equinox and the x-y plane in the equatorial plane of the earth to the required topocentric azimuth elevation system. This is accomplished by a series of coordinate transformations as follows:

1. A rotation of the coordinate system about the z-axis through an angle RA_s so that x-y plane is in the meridian plane of the station.

$$x' = x \cos RA_s + y \sin RA_s$$

$$y' = -x \sin RA_s + y \cos RA_s \qquad (I.1)$$

$$z' = z$$

The velocity transformation must take the rotational velocity of the new coordinate system into account.

$$\dot{x}' = y' \omega_e + \dot{x} \cos RA_s + \dot{y} \sin RA_s$$

$$\dot{y}' = -x' \omega_e - \dot{x} \sin RA_s + \dot{y} \cos RA_s$$

$$\dot{z}' = \dot{z}$$

where x', y', z' are the rotated coordinates and RA_s is the right ascension of the station and ω_e is the sidereal rate of the earth's rotation. The G.H.A. necessary to obtain RA_s from the station longitude is computed by the program.

2. A translation of the origin of the coordinate system from the center of the earth to the station in question

$$x'' = x' - (c + h)\cos\phi$$

$$y'' = y'$$

$$z'' = z' - (s + h)\sin\phi \qquad (I.2)$$

where

$$c = \left(1 - e^2 \sin^2 \phi\right)^{-1/2}$$

$$s = \left(1 - e^2\right) c$$

$$\dot{x}'' = \dot{x}' \ ; \quad \dot{y}'' = \dot{y}' \ ; \quad \dot{z}'' = \dot{z}'$$

where x'', y'', z'' are the translated coordinates. ϕ is the geodetic latitude and h the height above sea level of the station in question.

3. A rotation of $(90 - \phi)$ about the y'' axis to place the (x'', z'')

plane into the horizon plane

$$
\begin{aligned}
x''' &= x'' \sin\phi + z'' \cos\phi \\
y''' &= y'' \\
z''' &= -x'' \cos\phi + z'' \sin\phi \\
\dot{x}''' &= \dot{x}'' \sin\phi + \dot{z}'' \cos\phi \\
\dot{y}''' &= \dot{y}'' \\
\dot{z}''' &= -\dot{x}'' \cos\phi + \dot{z}'' \sin\phi
\end{aligned}
\tag{I.3}
$$

Now x''', y''', z''' are the coordinates of the vehicle in a topocentric azimuth elevation system, with z''' axis pointing to zenith and the x''' pointing south along the meridian. Range, range rate, azimuth and elevation are then given by

$$\rho = \left(x'''^2 + y'''^2 + z'''^2\right)^{1/2} = \text{Slant range} \tag{I.4}$$

$$\dot{\rho} = \frac{x''' \dot{x}''' + y''' \dot{y}''' + z''' \dot{z}'''}{\rho} = \text{Range rate} \tag{I.5}$$

$$E = \tan^{-1} \frac{z'''}{\left(x'''^2 + y'''^2\right)^{1/2}} = \text{Elevation} \tag{I.6}$$

$$A' = \tan^{-1}\frac{y'''}{x'''}$$

$$A = \begin{Bmatrix} \pi - A' & A' < \pi \\ 3\pi - A' & A' > \pi \end{Bmatrix} \tag{I.7}$$

J. TRIAXIAL MOON

Triaxial lunar potential constants (as used in the ITEM Program)

1. The values of the constants A, B and C for the perturbation accelerations due to the triaxial moon may be calculated using data from the NASA earth model meeting. These constants are currently being used in the ITEM Program.

2. The perturbation accelerations due to the triaxial moon are given by the partial derivatives of

$$\phi = \frac{C}{r^3}\left\{A\left(1 - \frac{3z^2}{r^2}\right) + B\left(1 - 3\frac{x^2}{r^2}\right)\right\} \qquad (J.1)$$

where

$$C = \frac{\mu_m a_m^2}{3 a_e^2}\left(\frac{3 I_C}{2 m a_m^2}\right)$$

$$A = \frac{I_C - I_A}{I_C} \qquad (J.2)$$

$$B = \frac{I_B - I_A}{I_C}$$

The form of the equations used in the ITEM Program are:

$$\frac{\partial \phi}{\partial x} = \frac{3xC}{r^5} F$$

$$\frac{\partial \phi}{\partial y} = \frac{3yC}{r^5} F - \frac{6CBy}{r^5} \qquad (J.3)$$

$$\frac{\partial \phi}{\partial z} = \frac{3zC}{r^5} F - \frac{6ACz}{r^5}$$

where

$$F = \left\{ A\left(\frac{5z^2}{r^2} - 1\right) + B\left(\frac{5y^2}{r^2} - 1\right) \right\}$$

Based on the NASA earth model meeting, the moments of inertia about the principal axes of the moon are:

$$I_A = .88746 \times 10^{35} \text{ kg meters}^2$$

$$I_B = .88764 \times 10^{35} \text{ kg meters}^2$$

$$I_C = .88801 \times 10^{35} \text{ kg meters}^2$$

(See figure 6)

Other constants are:

$$\mu_e = 19.9094165 \times \frac{(ER)^3}{m^2}$$

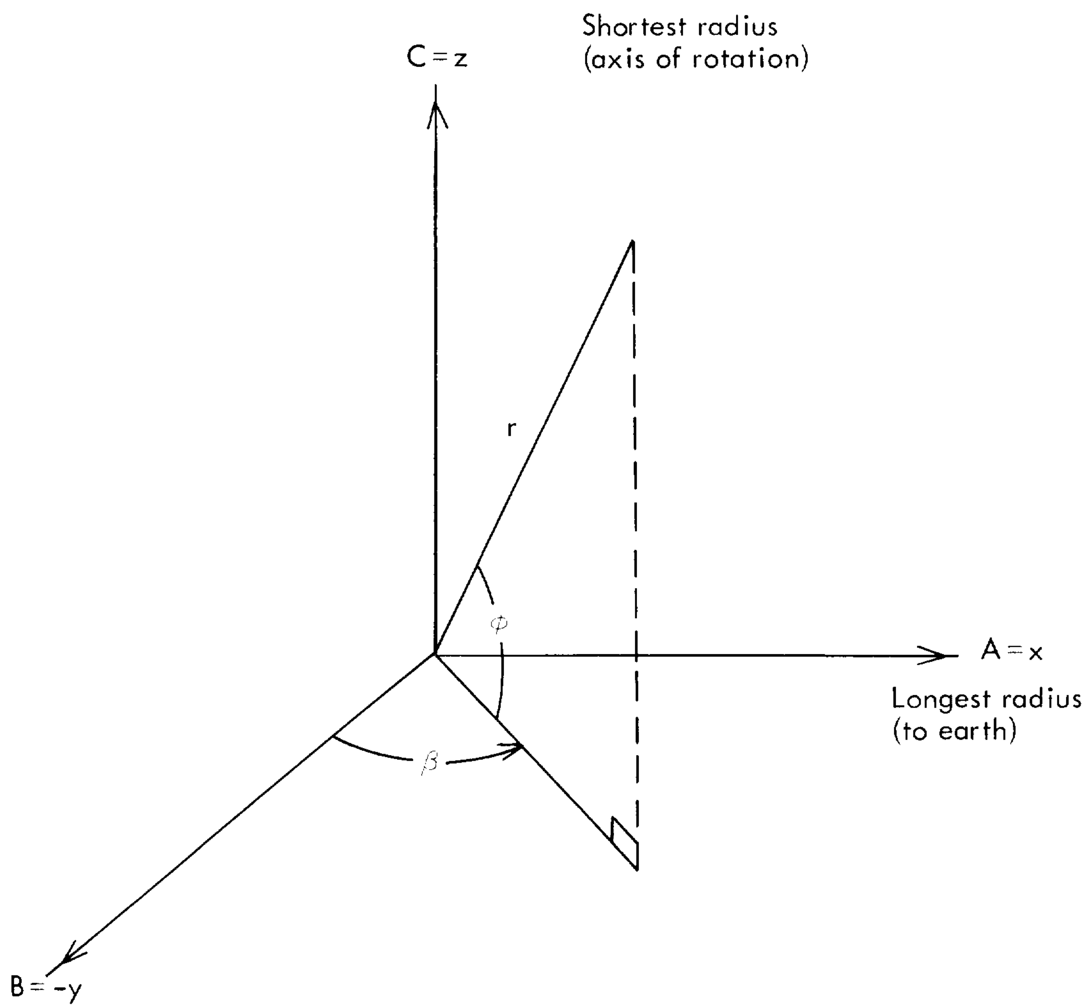


Figure 6. Triaxial Moon

$$\mu_m = \frac{\mu_e}{81.335} = .2447829 \times \frac{(ER)^3}{m^2}$$

$$\frac{(a_m)^2}{(a_\epsilon)} = \frac{(1738.09)^2}{(6378.165)} = .0742595 \text{ (earth radii)}^2$$

$$m_e = 5.975 \times 10^{27} \text{ grams}$$

$$m \equiv \frac{m_e}{81.335} = \frac{5.975 \times 10^{24}}{81.335} = 7.34616 \times 10^{22} \text{ kg}$$

$$a_m^2 = (1738.09)^2 = 3.0209568 \times 10^6 \text{ km}^2$$

The constants A, B and C may be calculated

$$A = \frac{I_C - I_A}{I_C} = \frac{(.88801 - .88746)}{.88801} = 619.36 \times 10^{-6}$$

$$B = \frac{I_B - I_A}{I_C} = \frac{(.88764 - .88746)}{.88801} = 202.70 \times 10^{-6}$$

$$C = \left(\frac{\mu_m a_m^2}{3 a_e^2}\right)\left(\frac{3 I_C}{2m a_m^2}\right)$$

For the ITEM Program the units of C are

$$\frac{(\text{earth radii})^5}{m^2}$$

therefore

$$C = \left\{\frac{\mu_m}{3}\left(\frac{a_M}{a_e}\right)^2\right\} \cdot \left(\frac{3 I_C}{2ma_m^2}\right)$$

$$= \left\{\frac{(.2447829)(.0742595)}{3}\right\}\left\{\frac{3(.88801) \times 10^{29}}{2(7.34616 \times 10^{22})(3.0209568 \times 10^8)}\right\}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

In summary the constants used in the ITEM Program as based upon the NASA earth model meeting are:

$$A = 619.36 \times 10^{-6}$$

$$B = 202.70 \times 10^{-6}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

K. DRAG COMPUTATION

The drag acceleration is computed, assuming a spherically symmetric atmosphere rotating with the earth. Thus:

$$D = -\frac{1}{2} \rho A \left| V_{eff} \right| V_{eff}$$

$$\Delta \ddot{R}_D = \frac{D}{m} \tag{K.1}$$

where

$$V_{eff} = \dot{R} - \omega \times R$$

ω is the sidereal rotation rate vector of the earth.

L. COMPUTATION OF SUBSATELLITE POINT

The geodetic coordinates of the subsatellite point are computed by the following method:

The geocentric latitude (declination) is obtained from

$$\sin \phi' = \frac{z}{r} \tag{L.1}$$

This latitude is then corrected to geodetic latitude by the formula

$$\phi = \phi' + a_2 \sin 2\phi' + a_4 \sin 4\phi' + a_6 \sin 6\phi' + a_8 \sin 8\phi' \tag{L.2}$$

where

$$a_2 = \frac{1}{1024r}\left\{512e^2 + 128e^4 + 60e^6 + 35e^8\right\}$$

$$+ \frac{1}{32r^2}\left\{e^6 + e^8\right\} - \frac{3}{256r^3}\left\{4e^6 + 3e^8\right\}$$

$$a_4 = -\frac{1}{1024r}\left\{64e^4 + 48e^6 + 35e^8\right\}$$

$$+ \frac{1}{16r^2}\left\{4e^4 + 2e^6 + e^8\right\} + \frac{15e^8}{256r^3} - \frac{e^8}{16r^4}$$

$$a_6 = \frac{3}{1024r}\left\{4e^6 + 5e^8\right\} - \frac{3}{32r^2}\left\{e^6 + e^8\right\}$$

$$+ \frac{35}{768r^3}\left\{4e^6 + 3e^8\right\} \tag{L.3}$$

$$a_8 = \frac{e^8}{2048}\left\{-\frac{5}{r}+\frac{64}{r^2}-\frac{252}{r^3}+\frac{320}{r^4}\right\}$$

e = the eccentricity of the earth

r = the distance from earth's center

See Reference 5.

The geodetic height is then given by

$$h = r\cos(\phi-\phi') - \sqrt{1-e^2\sin^2\phi} \tag{L.4}$$

The longitude is obtained by subtracting the sidereal time of Greenwich from the right ascension given by

$$\tan RA = \frac{y}{x} \tag{L.5}$$

M. POLAR COORDINATES REFERRED TO THE MOON

Moon longitude and latitude are defined in the coordinate system described in appendix J. If the vehicle coordinates with respect to the center of the moon in this coordinate system are given by x, y, z, r, then

$$\theta = \text{longitude} = \tan^{-1}\frac{y}{x}$$

$$\phi = \text{latitude} = \sin^{-1}\frac{z}{r}$$

N. SHADOW ROUTINE

The shadow calculations are in the process of reprogramming, therefore they will not be discussed in detail. It computes the umbra and penumbra shadow cones of the earth assuming a refraction angle of 68 minutes of arc and the radius of the sun 15'59''.63 at unit distance. The actual times of crossing the shadow boundaries are found by linear interpolation across the two nearest integration intervals.

O. SOLAR RADIATION PRESSURE

The radiation pressure subroutine computes the force of solar radiation on the spacecraft if an appropriate pressure coefficient is used. The calculation relies on the shadow routine to set a trigger to multiply the pressure coefficient by 1.0,0.5, or 0.0 for full sunlight, penumbra or umbra respectively. Therefore, the shadow subroutine must be used in conjunction with the radiation pressure routine for most cases. If the spacecraft is known to be continually in sunlight the number 1. may be loaded into SHADIN and thus elaborate shadow testing may be avoided.

$$P_{RP} = \frac{C_R \, A \, R_{VS}}{m \, r_{VS}^3} \tag{O.1}$$

(See Section VIII-H-4. for definition of symbols.)

This radiation pressure subroutine has been found to be inexact for satellites of large area to mass ratio since it only controls the pressure to the nearest integration step. For such spacecraft (e.g. balloons) several degrees error in true anomaly may result after 100 days unless the integration is carried exactly to the boundaries. A modification to achieve this increased precision is available and will be included in future versions of the program.

P. ECLIPTIC COORDINATES

The ecliptic coordinates are an approximate set obtained by a simple rotation of the equatorial coordinates about the x-axis through a fixed angle i = 23°26'31" which is approximately the true obliquity for Jan 0.0, 1962. More exact coordinates may be obtained by changing NE, (unit normal to the ecliptic) as desired.

Q. MOON ROTATING AND FIXED COORDINATE SYSTEM

Geocentric coordinates of the vehicle based on the earth-moon plane are generated from the geocentric equatorial radius vector to the vehicle, R_{VE}, the geocentric unit vector in the direction of the moon $\hat{R}_{ME}$, and the vector in the direction of the moon's velocity, $\dot{R}_{ME}$.

Coordinates in the rotating system, XROT etc., are found by using the current values of these vectors at each time step in the relations

$$\mathrm{XROT} = R_{VE} \cdot \hat{R}_{ME}$$

$$\mathrm{YROT} = R_{VE} \cdot \left(\hat{H}_{ME} \times \hat{R}_{ME} \right) \qquad (Q.1)$$

$$\mathrm{ZROT} = R_{VE} \cdot \hat{H}_{ME}$$

where

$$\hat{H}_{ME} = \frac{\hat{R}_{ME} \times \dot{R}_{ME}}{\left| \hat{R}_{ME} \times \dot{R}_{ME} \right|}$$

For the fixed axis system XINJ, etc., the initial vectors $\hat{R}_{ME}\left(t_0\right)$ and $\dot{R}_{ME}\left(t_0\right)$ at the time of injection are used with the current value of R_{VE}.

R. TRAJECTORY SEARCH

The program provides a search routine to obtain selected trajectories. The search is based on linear theory and varies the polar load input quantities (independent variables) to search for desired dependent variables. There are twelve possible dependent variables to select from although a maximum of seven of the twelve may be used in any given search. The quantities, at present, are

i, Ω, ω, t_p (pericenter time)

r_p (pericenter radius)

The above five variables at the moon, and at the earth (in the case of earth return trajectories) constitute the first ten variables. (normally referred to the equatorial plane - the earth-moon plane is also available. See Section VIII.)

In addition, the components of the impact parameter vector (B·T, B·R) may be selected. They are referred to the ecliptic plane for mars and venus trajectories and the moon's orbital plane for lunar trajectories. The number of independent variables must equal the number of dependent variables for this routine to operate. (See Input Options, Section IV-F).

The program integrates the nominal trajectory and each of the variational trajectories to obtain the required residuals and computes the matrix of partial derivatives. The required changes in initial conditions are then estimated linearly. The tolerance allowed on the dependent variables is to be specified, as is the number of iterations allowed. The printout at the end of each complete iteration contains the matrix, the residuals and the changes in the initial conditions. For convenience a 7 x 7 matrix is always printed, the significant elements will occur in the left upper corner. The numbers appearing in the printout of the matrix are the differences between the variation trajectories and the nominal, the first row being the differences of the first desired dependent variable with respect to each variation in turn.

The second row contains the differences in the second dependent variable etc. Each variation trajectory thus generates one column of the matrix.

This routine is time consuming if poor initial conditions are used. Two things should be done before it is used –

1. A first guess of the initial conditions of the nominal trajectory should be obtained from a patched conic or similar search program.

2. The number of variables should be kept to a minimum. It is planned to automate the iteration scheme in the future to go from two body, to patched conic, to full trajectory and to increase the number of variables to be adjusted, in optimal fashion. Even in its present form, however, it is extremely useful.

It is possible, through the use of the repeat case feature to cycle the variables being sought. It is possible to adjust the permissible size of the variable change by changing SIZER with a modification card.

S. EQUATIONS FOR FLIGHT PATH AZIMUTH AND FLIGHT PATH ANGLE

A subroutine computes the flight path azimuth and flight path angle with the following equations:

1. Flight path angle

$$\gamma = \sin^{-1}\left[\frac{\dot{R}}{V} \cdot \hat{N}\right] \tag{S.1}$$

$\hat{N}$ is the vertical unit vector. In the geodetic system $\hat{N}$ is given by

$$\hat{N} = \left[\cos\phi\cos\left(\theta - \theta_0\right), \cos\phi\sin\left(\theta - \theta_0\right), \sin\phi\right]$$

In the geocentric system ϕ is replaced by ϕ'. Alternatively, in the latter system

$$\hat{N} = \frac{R}{r}$$

2. Flight path azimuth

$$A = \sin^{-1}\left[\frac{1}{\cos\gamma}\left\{\frac{\dot{y}}{v}\cos\left(\theta - \theta_0\right) - \frac{\dot{x}}{v}\sin\left(\theta - \theta_0\right)\right\}\right]$$

$$A = \cos^{-1}\left[\frac{1}{\cos\gamma\cos\phi}\left\{\frac{\dot{z}}{v} - \sin\gamma\sin\phi\right\}\right] \tag{S.2}$$

Both formulas are used to determine the proper quadrant of A. To obtain the geocentric output, $e^2 = 0$, ϕ is replaced by declination $\delta = \phi'$.

T. OSCULATING ELEMENTS

The osculating elements are obtained from the following equations:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1} \tag{T.1}$$

$$n = \mu^{1/2}|a|^{-3/2} \tag{T.2}$$

$$\left.\begin{matrix} e \cos E \\ e \cosh E \end{matrix}\right\} = 1 - \frac{r}{a} \tag{T.3}$$

$$\left.\begin{matrix} e \sin E \\ e \sinh E \end{matrix}\right\} = \frac{d}{\sqrt{|\mu a|}} \tag{T.4}$$

$$M = \begin{cases} E - e \sin E \\ e \sinh E - E \end{cases} \tag{T.5}$$

$$t_p = t - \frac{M}{n} \tag{T.6}$$

The angles Ω, ω, i are obtained from the vectors H and $\hat{P}$, where

$$H = R \times \dot{R} \tag{T.7}$$

$$eP = \left(\frac{1}{r} - \frac{1}{a}\right) R - \frac{d}{\mu} \dot{R} \tag{T.8}$$

In terms of these vectors:

$$\cos i = \frac{H_z}{h} \qquad \text{in the first or fourth quadrant} \tag{T.9}$$

$$\sin \Omega = \frac{H_x}{h \sin i}$$

$$\cos \Omega = \frac{-H_y}{h \sin i} \tag{T.10}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega$$

$$\sin \omega = \frac{P_z}{\sin i} \tag{T.11}$$

U. IMPACT PARAMETERS

The "impact parameters" are coordinates in the "impact" plane. This plane passes thru the body (T-planet or the moon) and is normal to the incoming asymptote. The direction cosines of the asymptote are given by equations (U.1, U.2) in terms of unit vectors $\hat{P}$ (Appendix T) and

$$\hat{Q} = \frac{H}{h} \times \hat{P} \tag{U.1}$$

$$\hat{S} = \frac{1}{e}\left(\hat{P} + \sqrt{\left(e^2 - 1\right)}\hat{Q}\right) \tag{U.2}$$

In the plane defined by $\hat{S}$ as its normal, two unit vectors $\hat{T}_{IMP}$ and $\hat{R}_{IMP}$ are defined. $\hat{T}_{IMP}$ is parallel to the ecliptic plane for mars and venus impacts and to the moon's orbital plane for moon impacts. Explicitly

$$\hat{T}_{IMP} = \frac{\hat{N} \times \hat{S}}{|\hat{N} \times \hat{S}|} \tag{U.3}$$

where $\hat{N}$ is the unit normal to the ecliptic plane, or the moon's orbital plane. $\hat{R}_{IMP}$ is normal to both $\hat{S}$ and $\hat{T}_{IMP}$. B_{IMP} is the vector from the body to the vehicle as it crosses the impact plane. The information computed are the dot products

$$B_{IMP} \cdot \hat{T}_{IMP} \text{ and } B_{IMP} \cdot \hat{R}_{IMP}$$

V. MOON'S ORBITAL PLANE INPUT AND OUTPUT

A polar coordinate system is available for input and output which uses as its reference plane the moon's orbital plane and the vector from moon to earth as unit vector. Polar coordinates in this system are defined analogous to geocentric polar coordinates. The cartesian coordinates in this system are computed by equations (H.3) with

$$c = s = r_B$$

and

$$\theta_0 = 0$$

Here r_B is the radius of the body of departure (earth or moon).

These coordinates are then transformed to equatorial coordinates by a matrix C computed as follows:

$$\hat{i} = \frac{R_{EM}}{r_{EM}}$$

$$\hat{k} = \frac{R_{EM} \times \dot{R}_{EM}}{\left| R_{EM} \times \dot{R}_{EM} \right|} \qquad (V.1)$$

$$\hat{j} = \hat{k} \times \hat{i}$$

The transformation matrix C is then given by

$$C = \begin{pmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{pmatrix} \qquad (V.2)$$

and

$$\begin{aligned} R &= C R_{MOP} \\ \dot{R} &= C \dot{R}_{MOP} \end{aligned} \qquad (V.3)$$

The matrix C is unitary, and $C^{-1} = C^*$, permitting easy inversion of equations (V.2).

W. EQUATIONS FOR TRANSLUNAR PLANE INPUT

The translunar plane input is designed to permit easy visualization of the geometric relationships between initial conditions for circumlunar trajectories and the motion of the moon. (See Figure 1).

The initial conditions are given in a coordinate system referred to the translunar plane. This system has its x-axis along the ascending node of the vehicle with respect to the moon's orbital plane, its y-axis in the translunar plane, at right angles to the ascending node, in the direction of motion. In this coordinate system, initial position and velocity vectors are given by

$$x_{TL} = \left(r_B + h\right)\cos\Psi$$

$$y_{TL} = \left(r_B + h\right)\sin\Psi \qquad (W.1)$$

$$z_{TL} = 0$$

Here r_B is the radius of the body of departure (earth or moon).

$$\dot{x}_{TL} = v\sin(\gamma - \Psi)$$

$$\dot{y}_{TL} = v\cos(\gamma - \Psi) \qquad (W.2)$$

$$\dot{z}_{TL} = 0$$

The translunar plane is positioned by giving its inclination i_{TL} with respect to the moon's orbital plane and the lunar lead angle ϕ, the angle between the moon's position at injection and the descending node. The vectors R_{TL} and $\dot{R}_{TL}$ may then be transformed into the equatorial system by the following series of rotations:

1. A rotation $-i_{TL}$ about the x_{TL} axis will rotate the translunar plane into the moon's orbital plane.

2. A rotation of $\pi - (\lambda_M + \phi)$ about the new z-axis will refer the moon's orbital plane coordinate system to the ascending node of the moon's orbital plane (with respect to the equator) as x-axis.

Here λ_M stands for the argument of latitude of the moon. These rotations are performed by multiplying R_{TL} and $\dot{R}_{TL}$ by the matrix:

$$A = \begin{pmatrix} -\cos(\lambda_M + \phi) & \sin(\lambda_M + \phi) & -\sin(\lambda_M + \phi)\sin i_{TL} \\ -\sin(\lambda_M + \phi) & -\cos(\lambda_M + \phi)\cos i_{TL} & \cos(\lambda_M + \phi)\sin i_{TL} \\ 0 & \sin i_{TL} & \cos i_{TL} \end{pmatrix} \qquad \text{(W-3)}$$

3. The moon's orbital plane (MOP) is rotated about its node through an angle $-i_M$ (the inclination of the MOP).

4. The ascending node is brought into coincidence with the vernal equinox by a rotation $-\Omega_M$. These two rotations are embodied in the matrix

$$B = \begin{pmatrix} \cos\Omega_M & -\sin\Omega_M\cos i_M & \sin\Omega_M\sin i_M \\ \sin\Omega_M & \cos\Omega_M\cos i_M & -\cos\Omega_M\sin i_M \\ 0 & \sin i_M & \cos i_M \end{pmatrix} \qquad \text{(W-4)}$$

and thus:

$$R = (BA)\, R_{TL}$$

$$\dot{R} = (BA)\, \dot{R}_{TL} \qquad (W.5)$$